DRAFT

Unit of Study Proportional Relationships

Grade: 7

Topic: Unit Rates and Proportions

Focus of Learning

Common Core Standards:	Mathematical Practices:
Analyze proportional relationships and use them to solve real-world and mathematical	1. Make sense of problems and
problems.	persevere in solving them.
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths,	2. Reason abstractly and
areas and other quantities measured in like or different units. For example, if a person	quantitatively.
walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per	
hour, equivalently 2 miles per hour.	3. Construct viable arguments
 7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g. by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships c. Represent proportional relationships by equations. <i>For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.</i> d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. <i>Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees percent increase and decrease percent error</i>. 	 and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
fees, percent increase and decrease, percent error.	
Supporting Standards	
Draw, construct, describe, geometrical figures and describe the relationships	
between them.	
7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a	
different scale.	
Solve real-life and mathematical problems using numerical and algebraic expressions	
and equations.	
7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	
Enduring Understanding(s): Students will understand that	1
 A ratio or a rate expresses the relationship between two quantities. 	
 Ratio and rate reasoning can be applied to many different types of mathematical an 	id real-life problems.
 A ratio is a multiplicative comparison of two quantities. 	· · · · · · · · · · · · · · · · · · ·
 A proportion is a relationship of equality between two ratios. 	
5) A rate is a set of infinitely many equivalent ratios.	
6) Several ways of reasoning, all grounded in sense making, can be generalized into alg	porithms for solving proportion
problems.	,
Guiding Questions: These questions will guide student inquiry.	
 Why is unit rate important? 	
2) How are proportions used in everyday life?	
3) What kinds of problems can I solve with proportions?	
4) What kinds of problems can solve with proportional?	
5) When is it useful to be able to relate one quantity to another?	
6) How can I compare two different quantities?	
7) How are ratios, rates and proportions similar and different?	

- 8) What is a rate and how is it related to proportional reasoning?
- 9) When is it appropriate to reason proportionally?

Length of Unit: 22 – 28 Days

Student Performance

Student Ferrormance				
Knowledge: Students will understand/know	Application: Students will be able to			
 Unit rates associated with ratios of fractions can be measured in like or different terms Ratios and fractions do not have identical meanings; ratios are often used to make "part-to-part" comparisons, but fractions are not. The roles "for every", "for each," and "per" What creates a proportional relationship between two quantities The ratio of two quantities remains constant as the corresponding values of the quantities change proportionately. Effects of additive and multiplicative processes on proportionality Equivalent proportions take many forms, e.g. a/b = c/d is equivalent to a/c = b/d. Scale factor relates lengths amongst figures Areas do not scale by the same factor as lengths0 Slope, constant of proportionality, and unit rate represent the same value The slope of a line represents the constant of proportionality Lines that pass through the origin have a proportionality In y = cx, c represents the constant of proportionality In experimentation in the origin rate and proportionality A percent is a type of ratio that compares a quantity to 100 Appropriate use of proportional relationships to solve problems such as simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent 	 Identify unit rates in representations of proportional relationships Compute unit rates with rational numbers Make equivalent ratio tables of unit rates with complex fractions and decimals Identify relationships that are proportional in texts, graphs, tables, and equations Solve real-world problems involving proportions using tables of equivalent ratios Write and solve proportional equations, e.g. x/5 = 12/15 Identify the scale factor between two figures Compute actual lengths and areas from a scale drawings Reproduce a scale drawing at a different scale Identify the constant of proportionality from a table or a graph Graph proportional relationships and explain key features of the graph Communicate how the line continues from one point to the next (e.g. "every time the line goes 1 unit to the right, it goes up 3 units"). Use tables and graphs to write proportional relationships as equations, in the form y = cx, where c is the constant of proportional relationships to solve problems involving percent Use proportional relationships to solve problems involving percent Use proportional relationships to solve problems involving percent Compute simple interest, tax, gratuity, commission discount, markup, percent increase and decrease, and percent error 			
increase and decrease, and percent error.				

Assessments (Attached)

Pre-Assessment:

Formative Interim Assessment (Mid-Unit Checks):

- MARS—2001 Grade 7 "The Poster" (Lesson 6)
- Illustrative Mathematics: 7.RP "Robot Races" (Lesson 10)

Suggested Formative Assessments:

- Illustrative Mathematics: 7.RP "Art Class" (Use after Lesson 2)
- MARS—2005 Grade 7 "Lawn Mowing" (Use after Lesson 3 or Lesson 4)
- o Illustrative Mathematics: 7.G "Floor Plan" (Use after Lesson 5)
- Illustrative Mathematics: 7.RP "Tax and Tip" (Use after Lesson 12)
- MARS—2008 Grade 7 "Sale!" (Use after Lesson 13)

Post Assessment (Culminating Task):

• CORE Math_G7_Photos_FINAL_10_1012 "Photos"

	Learning Experiences (Lesson Plans Attached)			
Days	Lesson Sequence	<u>Materials</u>		
	 Lesson 1: Unit Rates with Like/Different Units (e.g. mph, people-hours, metrics) Students will know: Unit rates associated with ratios of fractions can be measured in like or different terms Ratios and fractions do not have identical meanings; ratios are often used to make "part-to-part" comparisons, but fractions are not. The roles of "for every", "for each," and "per" Students will be able to: Identify unit rates in representations of proportional relationships Compute unit rates from pairs of rational numbers Make equivalent ratio tables of unit rates with complex fractions and decimals 			
	 Lesson 2: Introduction to Proportions Students will know: What creates a proportional relationship between two quantities The ratio of two quantities remains constant as the corresponding values of the quantities change proportionately How additive and multiplicative processes effect proportionality Students will be able to: Identify relationships that are proportional in texts, graphs, tables, and equations 	 Suggested Formative Assessment: Illustrative Mathematics: 7.RP "Art Class" Van de Walle's <i>Teaching</i> <i>Student-Centered Mathematics,</i> <i>Grades 5-8</i> Activity 6.1 (pg. 158) 		
	 Lesson 3: Solve Proportions Using Tables of Equivalent Ratios Students will know: The roles "for every", "for each," and "per" The ratio of two quantities remains constant as the corresponding values of the quantities change proportionately. Students will be able to: Make tables of equivalent ratios Solve real-world problems involving proportions using tables of equivalent action 	 Suggested Formative Assessment: MARS—2005 Grade 7 "Lawn Mowing" 		
	equivalent ratios Lesson 4: Solving Proportions Using Equations Students will know: • The roles "for every", "for each," and "per" • The ratio of two quantities remains constant as the corresponding values of the quantities change proportionately. • Equivalent proportions take many forms, e.g. $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a}{c} = \frac{b}{d}$ Students will be able to:			
	 Write and solve proportional equations, e.g. x/5 = 12/15 Lesson 5: Scale Drawings Students will know: Scale factor relates lengths amongst figures Areas do not scale by the same factor as lengths Students will be able to: Identify the scale factor between two figures Compute actual lengths and areas from scale drawings 	Suggested Formative Assessment: Illustrative Mathematics: 7.G "Floor Plan" 		

r 12	
• Write and solve proportional equations, e.g. $\frac{x}{5} = \frac{12}{15}$	
 Solve real-world problems involving proportions using tables of equivalent ratios 	
 Reproduce a drawing at a different scale 	
Lesson 6: Review and Assessment	Interim Assessment:
Students will:	• MARS—2001 Grade 7 "The
Propose, justify and communicate a solution	Poster"
Lesson 7: Constant of Proportionality (Slope)	
Students will know:	
Slope, constant of proportionality, and unit rate represent the same value	
The ratio of two quantities remains constant as the corresponding	
values of the quantities change proportionately.	
Students will be able to:	
Identify the constant of proportionality from a table	
Lesson 8: Graphing Proportional Relationships	
Students will know:	
The slope of a line represents the constant of proportionality	
Lines that pass through the origin have a proportional relationship	
 In y = cx, c represents the constant of proportionality (slope) 	
 In the coordinate pair (1,r), r is the unit rate Since represents the vertical rate compared to (over the horizontal rate) 	
• Slope represents the vertical rate compared to/over the horizontal rate Students will be able to:	
 Identify relationships that are proportional on a graph 	
 Identify the constant of proportionality (i.e. slope/unit rate) from a graph 	
 Graph proportional relationships and explain key features of the graph 	
 Communicate how the line continues from one point to the next (e.g. 	
"every time the line goes 1 unit to the right, it goes up 3 units").	
Lesson 9: Represent Proportional Relationships as Equations (in the	
form y = cx)	
Students will know:	
 In y = cx, c represents the constant of proportionality (slope) 	
• In the coordinate pair (1,r), r is the unit rate	
Students will be able to:	
• Use tables and graphs to write proportional relationships as equations,	
in the form y = cx, where c is the constant of proportionality.	
• Employ a variety of variables into the basic equation y=cx (e.g. g=4q	
where g is gallons and q is quarts)	
Lesson 10: Review and Assessment	Interim Assessment:
Students will:	Illustrative Mathematics: 7.RP "Babat Bassa"
Propose, justify and communicate a solution	"Robot Races"
Lesson 11: Solve Percent Problems Using Proportions	Suggested Reference:
Students will know:	 Progressions for the Common
A percent is a type of ratio that compares a quantity to 100	Core State Standards in
Students will be able to:	Mathematics, Ratios and
Use proportional relationships to solve problems involving percents	Proportional Relationships Grades 6-8, pg. 11
Example: "8% of what number is 12?" can be solved using $\frac{8}{3} = \frac{12}{3}$	Gruues 0-0, pg. 11
Example: 8% of what humber is 12? can be solved using $\frac{100}{100} = \frac{1}{x}$	

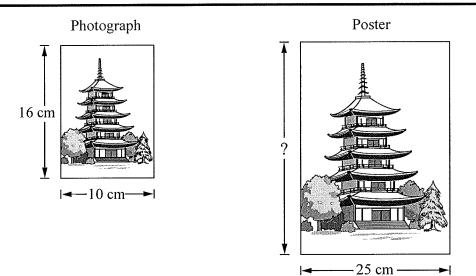
Le	esson 12: Problem Solving: Simple Interest, Ta	ax, Gratuity, and	Suggested Formative Assessment:
Co	ommission		Illustrative Mathematics: 7.RP
St	tudents will know:		"Tax and Tip"
•	Appropriate use of proportional relationships to	o solve problems such as	
	simple interest, tax, gratuities and commissions		
St	tudents will be able to:		
•	Represent a situation (as presented in a word p		
•	Use proportional relationships to solve real-wor	Id problems involving	
	ratios and percents		
•	Compute simple interest, tax, gratuity, and com		Suggested Formative Assessments
	esson 13: Problem Solving: Discount and Mar	кир	 Suggested Formative Assessment: MARS—2008 Grade 7 "Sale!"
	tudents will know:		• MARS—2008 Grade 7 Sale!
•	Appropriate use of proportional relationships to involving discounts and markups.	solve problems	
St	tudents will be able to:		
•	Represent a situation (as presented in a word p	roblem) as a proportion	
•	Use proportional relationships to solve real-wor	, , ,	
	ratios and percents		
•	Compute discount and markup		
Le	esson 14: Problem Solving: Percent Change ar	nd Percent Error	
St	tudents will know:		
•	Appropriate use of proportional relationships to	solve problems such as	
	percent increase and decrease, and percent erro	or	
St	tudents will be able to:		
•	Represent a situation (as presented in a word p		
•	Use proportional relationships to solve real-wor	Id problems involving	
	ratios and percents	areant arran	
•	Compute percent increase and decrease, and percession 15: Review	ercent error	
	tudents will:		
51	Propose, justify and communicate solutions		
	esson 16: Culminating Task		Summative Assessment:
	tudents will:		CORE "Photos"
•	Show their knowledge and understanding of un	it rates and proportions	
		ources	
	Online		Text
-	epartment of Education		achers of Mathematics.
	vw.georgiastandards.org/Common-		sential Understanding of Ratios,
Core/Pages	s/Math.aspx	•	Proportional Reasoning: Grades 6
		-	lational Council of teachers of
	e Mathematics	Mathematics,	inc. 2011.
<u>nttp://www</u>	w.illustrativemathematics.org/	Duenties Hall Martha	ation California Due Alecture
المعاملة معر	homotics (NAADS to de		atics. <i>California Pre-Algebra.</i>
	hematics/MARS tasks	Boston: Pearso	on Education, Inc. 2009.
	w.insidemathematics.org/;	Charaki Talma Math	matics Internationals Crade C
<u>nttp://map</u>	p.mathshell.org/materials/index.php	Shoseki, Tokyo. <i>Mathe</i> 2012. (Japanes	ematics International: Grade 6. Se Text)
Massachus	setts Department of Elementary and		
Secondary	Education	Van de Walle, John, ar	nd LouAnn Lovin. <i>Teaching</i>
http://www	w.doe.mass.edu/candi/model/units/	Student-Cente	red Mathematics: Grades 5-8. Vol.
		3. Boston: Pea	rson, 2006.
National Li	ibrary of Virtual Manipulatives		
http://nlvn	n.usu.edu/en/nav/vlibrary.html		

North Carolina Department of Public Instruction http://www.dpi.state.nc.us/acre/standards/common- core-tools/#unmath
Progressions for the Common Core State Standards in Mathematics http://ime.math.arizona.edu/progressions/
Smarter Balanced Assessment Consortium http://www.smarterbalanced.org/smarter-balanced- assessments/#item
Utah State Office of Education http://www.schools.utah.gov/CURR/mathsec/Core/7t h-Grade-Core/7RP.aspx

The Poster

This problem gives you the chance to:

• calculate sizes in an enlargement



A photograph is enlarged to make a poster.
 The photograph is 10 cm wide and 16 cm high.
 The poster is 25 cm wide. How high is the poster?
 Explain your reasoning.

2. On the poster, the building is 30 cm tall. How tall is it on the photograph? Explain your work.



	The Poster: Grade 7	points	section points
1.	Gives correct answer as:		
	40 cm.	1	
	Gives correct explanation that shows proportional reasoning such as: The poster is 2.5 times as wide as the photograph.		
	It must therefore be 2.5 times as high.	1	2
	Gives correct answer as:		
	12 cm	1	
	At least one of the correct answers shows the correct units.	1	
	Gives correct explanation that shows proportional reasoning such as:		
	The building is 30 cm tall on the poster.		
	We need to divide this measurement by 2.5 to find the height on the photograph.	1	3
	May show: $30 \div 2.5 = 12$ cm. Total Points		5

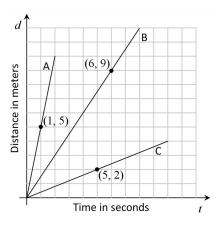
Illustrative Mathematics

7.RP Robot Races

Alignment 1: 7.RP.A.2

Carli's class built some solar-powered robots. They raced the robots in the parking lot of the school. The graphs below are all line segments that show the distance d, in meters, that each of three robots traveled after t seconds.

- a. Each graph has a point labeled. What does the point tell you about how far that robot has traveled?
- b. Carli said that the ratio between the number of seconds each robot travels and the number of meters it has traveled is constant. Is she correct? Explain.
- c. How fast is each robot traveling? How did you compute this from the graph?



Solution: Answers

- a. The point (1, 5) tells that robot A traveled 5 meters in 1 second. The point (6, 9) tells that robot B traveled 9 meters in 6 seconds. The point (5, 2) tells that robot C traveled 2 meters in 5 seconds.
- b. Carli is correct. Whenever the ratio between two quantities is constant, the graph of the relationship between them is a straight line through (0,0). We can also say that for each robot, the relationship between the time and distance is a proportional relationship.
- c. The speed can be seen as the *d*-coordinate of the graph when t = 1. This is the robot's unit rate: Robot A traveled 5 meters per second, as shown by the point (1, 5) on its graph. Robot B traveled 1.5 meters per second, as shown by the point (1, 1.5) on its graph. Robot C traveled 0.4 meters per second, as shown by the point (1, 0.4) on its graph.

The speed of each robot can also be seen in the steepness of its graph, which is quantified as slope. But that perspective is not expected until grade 8.



Student Name_

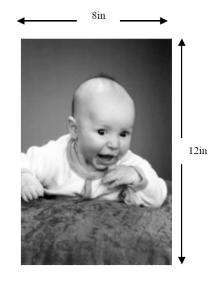
Photos

The *aspect ratio* of a picture is the ratio of its width to its height.

 a. Which of the following are correct expressions for the aspect ratio for this 8 inch by 12 inch baby photo? Draw rings around all of the correct ratios.

8:12	12: 8	4:6	2:3
3:2	1:1.5	16:24	24:36

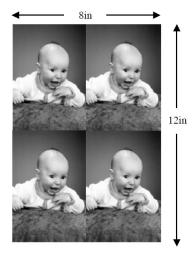
b. Choose one correct expression and explain why it is correct.



c. Choose one incorrect expression and explain why it is incorrect.

- When the length and width of the photo are halved, four photos fit on the 8-inch by 12-inch page. However, the aspect ratio is still the same as the original 8-inch by 12-inch photo.
 - a. What are the new measurements of each photo?
 - b. If you want 75 copies of this smaller size photo to use for holiday cards, how many 8-inch by 12inch pages would you need to print?

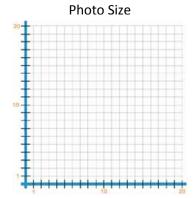
Show your work.



- 3. To use the photo on a key chain, the photo is reduced to a width of 1 inch, keeping the aspect ratio the same.
 - a. What is the height of the photo used for key chains?
 - b. Show and explain two different ways to find this height.

- c. What is the maximum number of key chain size photos that will fit on the 8-inch by 12-inch page?
- d. How do you know that this is the maximum number of photos? Write a convincing argument for your answer.

- 4. You can draw rectangles on a coordinate plane to represent the size of possible photos. The width and height of the photo can be represented using ordered pairs (width, height) such as (2, 3).
 - On a graph, plot the ordered pairs that represent each of the following three photo sizes. (Each size is given as width by height)
 - 2 inches by 3 inches
 - 4 inches by 6 inches
 - 8 inches by 12 inches



- b. Find the equation of the line through the plotted points.
- c. Draw the rectangles that represent the 3 photos on the graph. These photos have the same aspect ratio.
- d. What does the point (1,1.5) represent?

e. How could the point (1, 1.5) be used to quickly determine the height of a photo, if you are given that photo's width and the photo has the same aspect ratio as the photos in part (a)?

f. Jan claims that (5,7) represents a photo with the same aspect ratio. Explain whether she is correct or not. Show the work to support your explanation.

- 5. a. Determine three additional photo sizes that have the same aspect ratio as those in question 4a.
 - b. Predict how many additional rectangles could be drawn on the graph to represent photos with the same aspect ratio as those in question 4a. Explain your reasoning.

- 6. Most digital cameras capture images that are 600 pixels in width and 800 pixels in height.
 - a. What is the aspect ratio of a typical digital camera image?
 - b. When you order 4-inch by 6-inch prints of photos taken on a digital camera, the original images are often cropped. This means that parts along the edges of the original images are not included in the printed photos. Use aspect ratios to explain why this occurs.

c. What percent of the area of the original digital image is cropped to create a 4inch by 6-inch print?

Photos

Scoring Guide

Description	Points	Total Points
The core element of performance required by this tasks is:		
 Show understanding of ration and proportional relationships. 		
Based on this, credit for specific aspects of performance should be given as follows:		
1. a. Student gives correct answers with no extra ratios circled :	3	3
8:12 4:6 2:3 1:1.5 16:24 24:36		
Partial credit can be given for:		
 4 or 5 correctly circled with no extras or 6 correct and 1 extra 	(2)	
• 3 or 2 correctly circled with no extras or 4 to 5 correct and 1 extra	(1)	
b. Sample response should include statements such as:	1	2
 4:6 is a correct ratio because the ratio is "width to height" Dividing both the numerator and denominator by 2 gives a ratio equivalent to 8:12 	1	
Dividing both the numerator and denominator by 2 gives a ratio equivalent to 8:12 Sample response should include statements such as:	1	1
 c. Sample response should include statements such as: 12:8 or 3:2 is an incorrect ratio because the ratio used is "height to width" and the 	T	1
correct ratio should be "width to height"		
2. a. Student gives correct answer: 4 inches by 6 inches	1	4
b. Student gives correct answer: 19 pages	1	
Student shows work for 2a and 2b (This may include correct process but	2	
incorrect arithmetic)		
3. a. Student gives correct answer: 1.5 inches	1	1
b. Student shows work for two different methods, for example:		4
 Set up the ratio 8:12 and divide both numerator and denominator by 8 	1	
 Set up and solve the proportion 8/12 = 1/x 	1	
Explanation should include at least two of these " look for" phrases:		
• If I must divide the 8 by 8 to get 1 inch for the width, then I must divide the 12 by	1	
8 to get 1.5 inches for the height	1	
• Set up the proportion, cross-multiply, and solve for x		
 Set up the proportion, multiply both side by a common denominator and solve for x 		
c. Student gives the correct answer: The maximum number is 64 photos	1	1
d. Students write a convincing argument that might include:		3
• The area of each small picture is 1.5 in ² and the area of the paper is 96 in ² , so	3	
dividing 96 by 1.5 happens at most 64 times	or	
• Because the aspect ratio of the width of the age to the height of the page is	3	
equivalent to the aspect ratio of the width of the small photo to the height of the	0.5	
small photo, then the orientation of the photos to the paper will maintain the	or	
 aspect ratio. It will cover the whole paper and have no leftovers If the short side of every picture is oriented to the long side of the paper, you will 	1	
only get 60 photos and there will be unused space on the paper		
 You won't get as many photos because the aspect ratio would not be the same, 	1	
i.e., 8:12 versus 1.5:1		
 If your photos have mixed orientations, it is possible to again get 64 photos and 	1	
use up all of the paper; 64 in ² is the maximum area possible	1	1

Description	Points	Total Points
The core element of performance required by this tasks is:		
 Show understanding of ration and proportional relationships. 		
Based on this, credit for specific aspects of performance should be given as follows:		
4. a. Students correctly plot (2, 3), (4, 6), and (8, 12) on the coordinate graph	1 – 3	3
Give 1 point for each point plotted		
b. Student gives correct answer: y = 1.5x	1	1
c. Student correctly draws 3 rectangles with (0, 0) as a common vertex:		3
• First rectangle: (0, 0) as one vertex and (2, 3) as the diagonal vertex	1	
• Second rectangle: (0, 0) as one vertex and (4, 6) as the diagonal vertex	1	
• Third rectangle: (0, 0) as one vertex and (8, 12) as the diagonal vertex	1	
d. Student gives correct answer: It represent a photo that is 1 inch wide by 1.5 inches tall.	1	1
 e. Student correctly explain that you would multiply the width by 1.5 to determine the height. Student may also refer to the slope of the line through the diagonals of the rectangles (1.5/1) to justify their reasoning. Give 1 point for a limited response or 2 points for a complete response 	1 - 2	2
f. Student gives correct answer: Jan is not correct.	1	3
Student correctly explains why Jan is incorrect, for example:		
• Using the cross product with 8/12 = 5/7 gives 56 = 60; because they aren't equal,	2	
they are not proportional, so the aspect ratio is not the same.		
 A geometric explanation would be that the point (5, 7) is not on the same diagonal as the other photos with an aspect ratio of 8:12 		
5. a. Sample responses may include: (6, 9) (10, 15) (12, 18) (1.5, 2.25) etc	1	3
b. Student gives correct answer: infinitely many	1	3
Student explanation should include:	1 – 2	
• You can get an infinite number of rectangles with the same aspect ratio of 2:3 by		
doing the following: for any width that you are given, as long as you multiply the		
width by 1.5 you will generate the corresponding height.		
Give 1 point for a limited response or 2 points for a complete response		
6. a. Student gives correct answer: 800:600 or 2:3	1	1
b. Student correctly explains that the two ratios are not equal, which accounts for part	1 – 2	2
of the photo being cropped off when it is developed.		
Give 1 point for a limited response or 2 points for a complete response		
c. Student gives correct answer: 12% of the photo has been cropped	1 – 2	2
TOTAL POINTS		43
(Possible points =)		