Sacramento City Unified School District

CCSS-M Teacher Professional Learning

Session #1, October 2014

Grade 8

Packet Contents

(Selected pages relevant to session work)

Content Standards

Standards for Mathematical Practice

California Mathematical Framework

Kansas CTM Flipbook

Learning Outcomes

Sample Assessment Items

8 Grade 8

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
    b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.

Common Core State Standards - Mathematics
Standards for Mathematical Practices – 8th Grade

Standard for Mathematical Practice	8th Grade
1: Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, -What is the most efficient way to solve the problem?, -Does this make sense?, and -Can I solve the problem in a different way?

2: Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3: Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like -How did you get that?, -Why is that true? -Does that always work? They explain their thinking to others and respond to others' thinking.

4: Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

5: Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
6: Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.

7: Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
8: Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

- Taking the square root of the square of a number *sometimes* returns the number back (e.g., $\sqrt{7^2} = \sqrt{49} = 7$, while $\sqrt{(-3)^2} = \sqrt{9} = 3 \neq -3$)
- Cubing a number and taking the cube root can be considered inverse operations.

Students expand their exponent work as they perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students use scientific notation to express very large or very small numbers. Students compare and interpret scientific notation quantities in the context of the situation, recognizing that the powers of ten indicated in quantities expressed in scientific notation follow the rules of exponents shown above. (Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina 2013)

Example: Ants and Elephants. An ant has a mass of approximately 4×10^{-3} grams and an elephant has a mass of approximately 8 metric tons. How many ants does it take to have the same mass as an elephant?

(Note: 1 kg = 1000 grams, 1 metric ton = 1000 kg.)

Solution: To compare the masses of an ant and an elephant, we convert the mass of an elephant into grams:

$$8 \text{ metric tons} \times \frac{1000 \text{ kg}}{1 \text{ metric ton}} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 8 \times 10^3 \times 10^3 \text{grams} = 8 \times 10^6 \text{grams}.$$

If we let N represent the number of ants that have the same mass as an elephant, then $(4 \times 10^{-3})N$ is their total mass in grams. This should equal 8×10^6 grams. This gives us a simple equation:

$$(4 \times 10^{-3})N = 8 \times 10^6 \text{ which means that } N = \frac{8 \times 10^6}{4 \times 10^{-3}} = 2 \times 10^{6-(-3)} = 2 \times 10^9$$

Thus, 2×10^9 ants would have the same mass as an elephant.

(Adapted from Illustrative Mathematics, 8.EE Ant and Elephant.)

[Note: Sidebar]

Focus, Coherence, and Rigor:

As students work with scientific notation, they learn to choose units of appropriate size for measurement of very large or very small quantities. (**MP.2, MP.5, MP.6**)

Expressions and Equations	8.EE
Understand the connections between proportional relationships, lines, and linear equations.	
5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two	

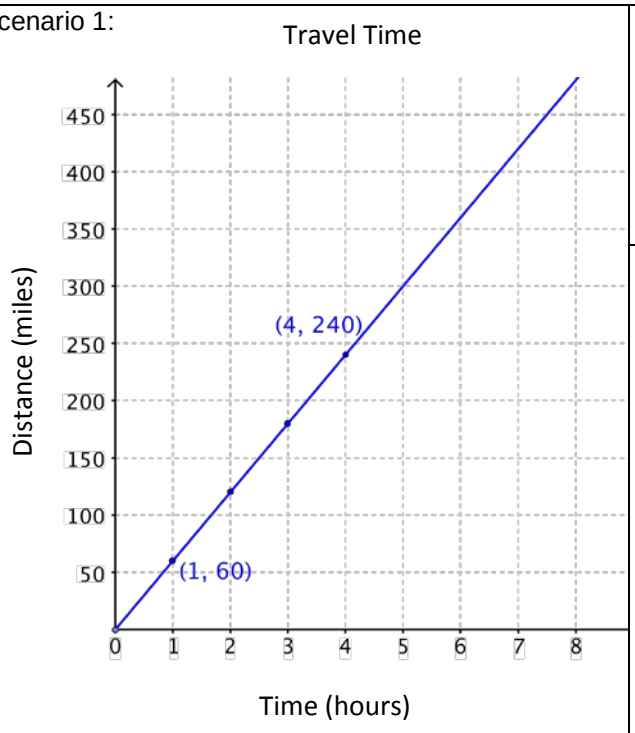
different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation and determine which of the two moving objects has greater speed.*

6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at *b*.

Students build on their work with unit rates from sixth grade and proportional relationships in seventh grade to compare graphs, tables, and equations of proportional relationships (**8.EE.5▲**). Students identify the unit rate (or slope) to compare two proportional relationships represented in different ways (e.g., as graph of the line through the origin, a table exhibiting a constant rate of change, or an equation of the form $y = kx$). Students interpret the unit rate in a proportional relationship (e.g., r miles per hour) as the slope of the graph. They understand that the slope of a line represents a constant rate of change.

Example: Compare the scenarios below to determine which represents a greater speed. Include a description of each scenario that discusses unit rates in your explanation.

Scenario 1:



Scenario 2: The equation for the distance y in miles as a function of the time x in hours is:

$$y = 55x.$$

Solution: "The unit rate in Scenario 1 can be read from the graph; it is 60 miles per hour. In Scenario 2, I can see that this looks like an equation $y = kx$, and in that type of equation the unit rate is the constant k. Therefore the speed in Scenario 2 is 55 miles per hour. So the person traveling in Scenario 1 is going faster."

(Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina 2013)

Following is an example of connecting the Standards for Mathematical Content with the Standards for Mathematical Practice.

211 **Connecting to the Standards for Mathematical Practice—Grade Eight**

<table>
<tr><th>Standard(s) Addressed</th><th>Example(s) and Explanations</th></tr>
<tr><td>8.EE.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in two different ways.</td><td>
Task: Below is a table that shows the costs of various amounts of almonds.
<table>
<tr><td>Almonds (pounds)</td><td>3</td><td>5</td><td>8</td><td>10</td><td>15</td></tr>
<tr><td>Cost (dollars)</td><td>15.00</td><td>25.00</td><td>40.00</td><td>50.00</td><td>75.00</td></tr>
</table>
1. Graph the cost versus the number of pounds of almonds. The number of pounds of almonds should be on the horizontal axis and the cost of the almonds on the vertical axis.
2. Use the graph to find the cost of 1 pound of almonds. Explain how you got your answer.
3. The table shows that 5 pounds of almonds costs $25.00. Use your graph to find out how much 6 pounds of almonds costs.
4. Suppose that walnuts cost $3.50 per pound. Draw a line on your graph that might represent the cost of different numbers of pounds of walnuts.
5. Which is cheaper? Almonds or walnuts? How do you know?
Solution:
1. A graph is shown.
2. To find the cost of 1 pound of almonds, one would locate the point that has first coordinate 1; this is the point (1, 5). This shows that the unit cost is $5 per pound.
3. Students can do this by simply locating 6 pounds on the horizontal axis and finding the point on the graph associated with this number of pounds. However, the teacher can also urge students to notice that one can move along the graph by moving to the right 1 unit and noticing that we move 5 units up to the next point on the graph. This idea is the genesis of slope of a line and should be explored.
4. Ideally, students draw a line that passes through (0,0) and the approximate point (1, 3.50). Proportional thinkers might notice that 2 pounds of walnuts cost $7, so they can plot a point with whole number coordinates.
5. Walnuts are cheaper. Students can explore several different ways to see this, including the unit cost, the steepness of the line, by comparing common quantities of nuts, etc.
Classroom Connections
The concept of slope can be approached in its simplest form with directly proportional quantities. In this case, when two quantities x and y are directly proportional, they are related by an equation $y = kx$, equivalently, $\frac{y}{x} = k$, where k is a constant known as the constant of proportionality. In the case of almonds above, the k in an equation would represent the unit cost of almonds. Students should have several experiences with graphing and exploring directly proportional relationships to build a foundation for understanding more general linear equations of the form $y = mx + b$.
Connecting to the Standards for Mathematical Practice
(MP.1) Students are encouraged to attack the entire problem and make sense in each step required.
(MP.4) Students are modeling a very simple real-life cost situation.
</td></tr>
</table>

212

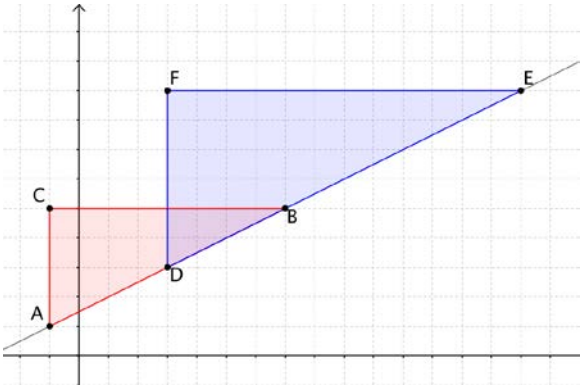
[Note: Sidebar]

Focus, Coherence, and Rigor:

The connection between the unit rate in a proportional relationship and the slope of its graph depends on a connection with the geometry of similar triangles. (See Standards **8.G.4-5▲**.) The fact that a line has a well-defined slope—that the ratio between the rise and run for any two points on the line is always the same—depends on similar triangles (Adapted from Progressions 6-8 EE 2011).

Standard **(8.EE.6▲)** represents a convergence of several ideas in this and previous grade levels. Students have graphed proportional relationships and found the slope of the resulting line, interpreting it as the unit rate **(8.EE.5▲)**. It is here that the language of "rise over run" comes into use. In the Functions domain, students will see that any linear equation $y = mx + b$ determines a function whose graph is a straight line (a *linear function*), and they verify that the slope of the line is equal to m (**8.F.3**). In standard **(8.EE.6▲)**, students go further and explain why the slope m is the same through any two points on a line. They justify this fact using similar triangles, which are studied in standards **(8.G.4-5▲)**.

239

<table>
<tr><td colspan="2">Example of Reasoning (8.EE.6 ▲). Showing that the slope is the same between two points on a line.</td></tr>
<tr><td>In seventh grade, students made scale drawings of figures and observed the proportional relationships between side lengths of such figures (7.G.1▲). In grade eight, students generalize this idea and study dilations of plane figures, and they define figures as being similar in terms of dilations (see standard 8.G.4▲). It is discovered that similar figures share a proportional relationship between side lengths just like scale drawings did: there is a scale factor $k > 0$ such that corresponding side lengths of similar figures are related by the equation $s_1 = k \cdot s_2$. Furthermore, the ratio of two sides in one shape is equal to the ratio of the corresponding two sides in the other shape. Finally, in standard (8.G.5▲), students informally argue that triangles that have two corresponding angles of the same measure must be similar, and this is the final piece of the puzzle for the first result in standard (8.EE.6▲).</td><td>Example: “Explain why the slope between points A and B and points D and E are the same.”
Solution: “Angles $\angle A$ and $\angle D$ are equal since they are corresponding angles formed by the transversal crossing the vertical lines through points A and D. Since $\angle C$ and $\angle F$ are both right angles, the triangles are similar. This means the ratios $\frac{AC}{BC}$ and $\frac{DF}{EF}$ are equal. But when you find the ‘rise over the run,’ these are exactly the ratios that you find, and so the slope is the same between these two sets of points.”
 
</td></tr>
</table>

240

241 In grade eight students build on previous work with proportional relationships,
242 unit rates, and graphing to connect these ideas and understand that the points
243 (x, y) on a non-vertical line are the solutions of the equation $y = mx + b$, where
244 m is the slope of the line, as well as the unit rate of a proportional relationship in
245 the case $b = 0$.

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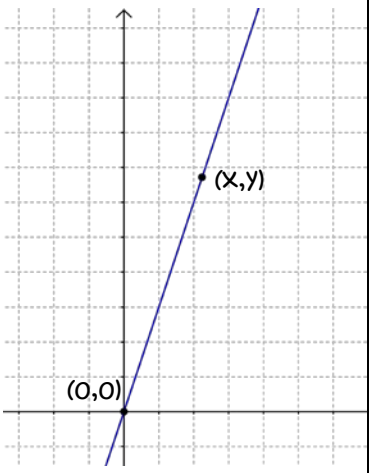
Example of Reasoning (8.EE.6). Deriving the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Example: "Explain how to derive the equation $y = 3x$ for the line of slope $m = 3$ shown below." **Solution: "**I know that the slope is the same between any two points on a line. So I'll choose the origin $(0,0)$ and a generic point on the line, calling it (x, y). By choosing a generic point like this I know that any point on the line will fit the equation I come up with. The slope between these two points is found by



$$3 = \frac{\text{rise}}{\text{run}} = \frac{y-0}{x-0} = \frac{y}{x}$$

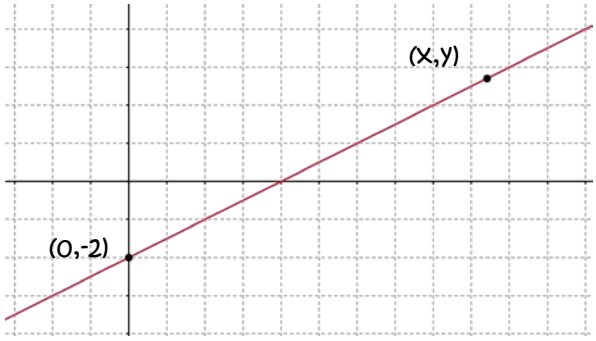
This equation can be rearranged to $y = 3x$."

Example: "Explain how to derive the equation $y = \frac{1}{2}x - 2$ for the line of slope $m = \frac{1}{2}$ with intercept $b = -2$ shown below."

Solution: "I know the slope is $\frac{1}{2}$ so I'll calculate the slope using the point $(0, -2)$ and the generic point (x, y). The slope between these two points is found by

$$\frac{1}{2} = \frac{\text{rise}}{\text{run}} = \frac{y-(-2)}{x-0} = \frac{y+2}{x}$$

This can be rearranged to $y + 2 = \frac{1}{2}x$, which is the same as $y = \frac{1}{2}x - 2$."



(Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina 2013)

Expressions and Equations	8.EE

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Domain: **Expressions and Equations (EE)**

Cluster: Understand the connections between proportional relationships, lines, and linear equations

Standard: **8.EE.5.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Standards for Mathematical Practice (MP):

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the Grade 8 Critical Area of Focus #1, **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equaitons** and Critical Area of Focus #3, **Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem**.

Explanations and Examples

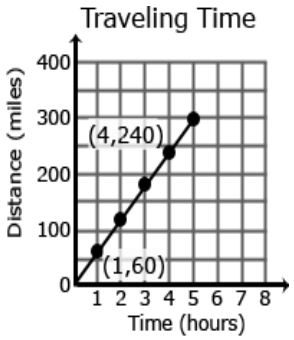
8.EE.5 Students build on their work with unit rates from 6 grade and proportional relationships in 7 grade to compare graphs, tables and equations of proportional relationships. Students identify the unit rate (or slope) in graphs, tables and equations to compare two or more proportional relationships.

Using graphs of experiences that are familiar to students increases accessibility and supports understanding and interpretation of proportional relationship. Students are expected to both sketch and interpret graphs.

Example:

- Compare the scenarios to determine which represents a greater speed. Include a description of each scenario including the unit rates in your explanation.

Scenario 1:



Scenario 2:

$y = 50x$
x is time in hours
y is distance in miles

Instructional Strategies

This cluster focuses on extending the understanding of ratios and proportions. Unit rates have been explored in Grade 6 as the comparison of two different quantities with the second unit a unit of one, (unit rate). In seventh grade unit rates were expanded to complex fractions and percents through solving multistep problems such as: discounts, interest, taxes, tips, and percent of increase or decrease. Proportional relationships were applied in scale drawings, and students should have developed an informal understanding that the steepness of the graph is the slope or unit rate. Now unit rates are addressed formally in graphical representations, algebraic equations, and geometry through similar triangles.

Distance time problems are notorious in mathematics. In this cluster, they serve the purpose of illustrating how the rates of two objects can be represented, analyzed and described in different ways: graphically and algebraically. Emphasize the creation of representative graphs and the meaning of various points. Then compare the same information when represented in an equation. By using coordinate grids and various sets of three similar triangles, students can prove that the slopes of the corresponding sides are equal, thus making the unit rate of change equal. After proving with multiple sets of triangles, students can be led to generalize the slope to $y = mx$ for a line through the origin and $y = mx + b$ for a line through the vertical axis at b.

Instructional Resources/Tools

Carnegie Math™
graphing calculators
SMART™ technology with software emulator
National Library of Virtual Manipulatives (NLVM)©,
The National Council of Teachers of Mathematics, Illuminations
Annenberg™ video tutorials, www.nsdl.org

Domain: Expressions and Equations

Cluster: Understand the connections between proportional relationships, lines, and linear equations.

Standard: **8.EE.6.** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

Standards for Mathematical Practice (MP):

MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:

See **8.EE.5.**

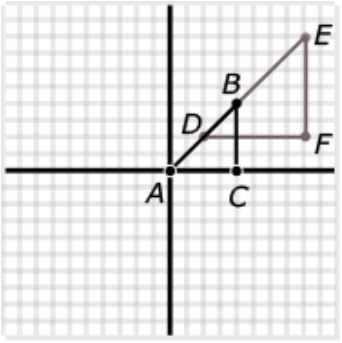
Explanations and Examples

8.EE.6 Triangles are similar when there is a constant rate of proportion between them. Using a graph, students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line.
The triangle between A and B has a vertical height of 2 and a horizontal length of 3. The triangle between B and C has a vertical height of 4 and a horizontal length of 6. The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents a slope of 2/3for the line.

Students write equations in the form y = mx for lines going through the origin, recognizing that m represents the slope of the line. Students write equations in the form y = mx + b for lines not passing through the origin, recognizing that m represents the slope and b represents the y-intercept.

Example:

- Explain why ΔACB is similar to ΔDFE, and deduce that $\overline{AB}$ has the same slope as $\overline{BE}$. Express each line as an equation.



Instructional Strategies

See **8.EE.5.**

Common Misconceptions:

See **8.EE.5.**

SCUSD 8th Grade Curriculum Map

Unit 3: Linear Relationships	
Sequence of Learning Outcomes **8.EE.5, 8.EE.6, 8.F.2**	
1)	Graph proportional relationships given a real-world context and interpret the unit rate as the slope of the graph. 8.EE.5
2)	Compare two different proportional relationships represented in different ways, for example, in a graph, a table, an equation, and a verbal description. 8.EE.5, 8.F.2
3)	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (Framework p. 16) 8.EE.6
4)	Derive and understand slope/rate of change given a real-world context by using graphs, tables, equations (y=mx) and verbal descriptions in the first quadrant. 8.EE.5, 8.EE.6
5)	Derive and understand slope/rate of change with a y-intercept given a real-world context by using graphs, tables, equations (y=mx + b) and verbal descriptions in the first quadrant. 8.EE.6
6)	Derive and understand slope/rate of change and y-intercept in context in all quadrants. 8.EE.6
7)	Model real-world problems with the relationships y=mx and y=mx + b. Determine what parts of the graph make sense in context of the situation. 8.EE.5, 8.EE.6

Big Ideas Math Course 3

Chapter 4: Graphing and Writing Linear Equations
Sequence of Learning Objectives **Lessons 4.1 – 4.5**
Lesson 4.1 – Graphing Linear Equations In this lesson, you will • Understand that lines represent solutions of linear equations • Graph linear equations Preparing for Standard 8.EE.5
Lesson 4.2 – Slope of a Line In this lesson, you will • Find slopes of lines by using two points • Find slopes of lines from tables (*Note:* This lesson includes 1 activity in which students use similar triangles to understand slope, but no corresponding problems in the "Practice and Problem Solving" section.) 8.EE.6
Lesson 4.3 – Graphing Proportional Lines In this lesson, you will • Write and graph proportional relationships (*Note:* This lesson includes 1 activity on comparing different proportional relationships represented in different ways, with a few corresponding practice problems.) 8.EE.5, 8.EE.6
Lesson 4.4 – Graphing Linear Equations in Slope-Intercept Form In this lesson, you will • Find slopes and y-intercepts of graphs of linear equations • Graph linear equations written in slope-intercept form 8.EE.6
Lesson 4.5 – Graphing Linear Equations in Standard Form In this lesson, you will • Graph linear equations written in standard form Applying Standard 8.EE.6

Sample Assessment Questions
8.EE.5 and 8.EE.6

From www.IllustrativeMathematics.org (linked from SCUSD Curriculum Map)

1. Coffee By the Pound

Lena paid $18.96 for 3 pounds of coffee.

a. What is the cost per pound for this coffee?

b. How many pounds of coffee could she buy for $1.00?

c. Draw a graph in the coordinate plane of the relationship between the number of pounds of coffee and the total cost.

d. In this situation, what is the meaning of the slope of the line you drew in part (c)?

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2. Stuffing Envelopes

Anna and Jason have summer jobs stuffing envelopes for two different companies. Anna earns $14 for every 400 envelops she finishes. Jason earns $9 for every 300 envelopes he finishes.

a. Draw graphs and write equations that show the earnings, y as functions of the number of envelopes stuffed, n for Anna and Jason.

b. Who makes more from stuffing the same number of envelopes? How can you tell this from the graph?

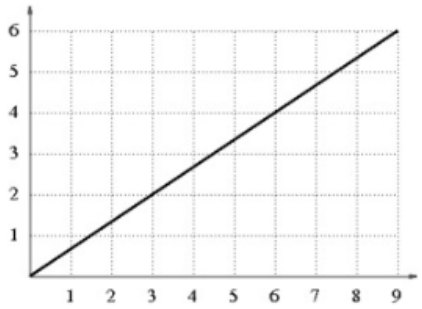
c. Suppose Anna has savings of $100 at the beginning of the summer and she saves all her earnings from her job. Graph her savings as a function of the number of envelopes she stuffed, n. How does this graph compare to her previous earnings graph? What is the meaning of the slope in each case?

3. Slopes Between Points on a Line

The slope between two points is calculated by finding the change in y-values and dividing by the change in x-values. For example, the slope between the points (7, -15) and (-8, 22) can be computed as follows:

- The difference in the y-values is $-15 - 22 = -37$.
- The difference in the x-values is $7 - (-8) = 15$.
- Dividing these two differences, we find that the slope is $-\frac{37}{15}$.

Eva, Carl, and Maria are computing the slope between pairs of points on the line shown below.



Eva finds the slope between the points (0,0) and (3,2). Carl finds the slope between the points (3,2) and (6,4). Maria finds the slope between the points (3,2) and (9,6). They have each drawn a triangle to help with their calculations (shown below).

a. Which student has drawn which triangle? Finish the slope calculation for each student. How can the differences in the x- and y-values be interpreted geometrically in the pictures they have drawn?

b. Consider any two points (x_1, y_1) and (x_2, y_2) on the line shown above. Draw a triangle like the triangles drawn by Eva, Carl, and Maria. What is the slope between these two points? Why should this slope be the same as the slopes calculated by the three students?

4 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Find the slope and the *y*-intercept of the graph of the linear equation.

1. $y = 6x - 5$

2. $y = 20x + 15$

3. $y = -5x - 16$

4. $y - 1 = 3x + 8.4$

5. $y + 4.3 = 0.1x$

6. $-\frac{1}{2}x + 2y = 7$

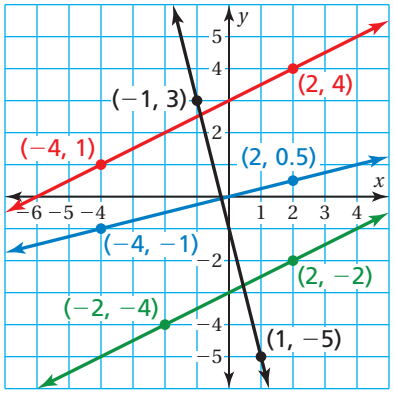
Graph the linear equation.

7. $y = 2x + 4$

8. $y = -\frac{1}{2}x - 5$

9. $-3x + 6y = 12$

10. Which lines are parallel? Which lines are perpendicular? Explain.

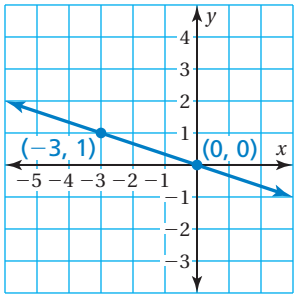


11. The points in the table lie on a line. Find the slope of the line.

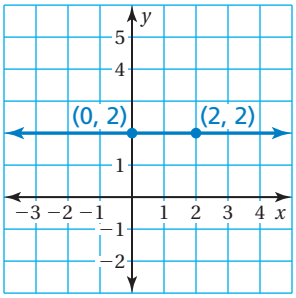
x	*y*
−1	−4
0	−1
1	2
2	5

Write an equation of the line in slope-intercept form.

12.



13.



Write in slope-intercept form an equation of the line that passes through the given points.

14. $(-1, 5), (3, -3)$

15. $(-4, 1), (4, 3)$

16. $(-2, 5), (-1, 1)$

17. **VOCABULARY** The number y of new vocabulary words that you learn after x weeks is represented by the equation $y = 15x$.

a. Graph the equation and interpret the slope.

b. How many new vocabulary words do you learn after 5 weeks?

c. How many more vocabulary words do you learn after 6 weeks than after 4 weeks?

Test Item References

Chapter Test Questions	Section to Review	Common Core State Standards
7, 8	4.1	8.EE.5
10, 11	4.2	8.EE.6
17	4.3	8.EE.5, 8.EE.6
1–6	4.4	8.EE.6
9	4.5	8.EE.6
12, 13	4.6	8.F.4
14–16	4.7	8.F.4

Test-Taking Strategies

Remind students to quickly look over the entire test before they start so that they can budget their time. Students should jot down the formulas for slope-intercept form and point-slope form on the back of their test before they begin. Teach students to use the Stop and Think strategy before answering. **Stop** and carefully read the question, and **Think** about what the answer should look like.

Common Errors

- **Exercises 1–6** Students may use the reciprocal of the slope when graphing and may find an incorrect *x*-intercept. Remind them that slope is *rise* over *run*, so the numerator represents vertical change, not horizontal.
- **Exercises 7–9** Students may make calculation errors when solving for ordered pairs. If they only find two ordered pairs for the graph, they may not recognize their mistakes. Encourage them to find at least three ordered pairs when drawing a graph.
- **Exercise 12** Students may write the reciprocal of the slope or forget a negative sign. Ask them to predict the sign of the slope based on the rise or fall of the line.
- **Exercise 14–16** Students may use the reciprocal of the slope when writing the equation. Remind them that slope is the change in *y* over the change in *x*.

Reteaching and Enrichment Strategies

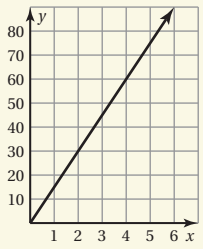
If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials *BigIdeasMath.com* Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start Standards Assessment

Answers

1. slope: 6; y-intercept: -5
2. slope: 20; y-intercept: 15
3. slope: -5; y-intercept: -16
4. slope: 3; y-intercept: 9.4
5. slope: 0.1; y-intercept: -4.3
6. slope: $\frac{1}{4}$; y-intercept: $\frac{7}{2}$

7–9. See Additional Answers.

10. The red and green lines are parallel. They both have a slope of $\frac{1}{2}$. The black and blue lines are perpendicular. The product of their slopes is -1.
11. 3
12. $y = -\frac{1}{3}x$
13. $y = 2$
14. $y = -2x + 3$
15. $y = \frac{1}{4}x + 2$
16. $y = -4x - 3$
17. **a.**



You learn 15 new vocabulary words per week.

b. 75 new vocabulary words

c. 30 more words

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