Sacramento City Unified School District

CCSS-M Teacher Professional Learning

Session #1, October 2014

Grade 5

Packet Contents

(Selected pages relevant to session work)

5 Grade 5

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

2.1 Express a whole number in the range 2–50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as 2x2x2x3. CA

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Number and Operations in Base Ten 5.NBT

Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Common Core State Standards - Mathematics
Standards for Mathematical Practices – 5th Grade

Standard for Mathematical Practice	5th Grade
1: Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, -What is the most efficient way to solve the problem?, -Does this make sense?, and -Can I solve the problem in a different way?

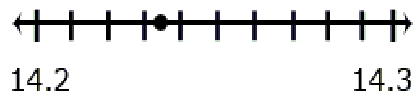
2: Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
3: Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	In fifth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like -How did you get that? and -Why is that true? They explain their thinking to others and respond to others’ thinking.

4: Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
5: Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose	Fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.

6: Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
7: Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	In fifth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

8: Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

just focus on rounding 235 (thousandths) to the nearest hundred. In that case, since 235 would round down to 200, we'd get 14.200.



Students can use benchmark numbers (e.g., 0, 0.5, 1, and 1.5) to support similar work.

Number and Operations in Base Ten 5.NBT

Perform operations with multi-digit whole numbers and with decimals to hundredths.

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7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties or operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

In grades three and four, students used various strategies to multiply. In grade five students fluently multiply multi-digit whole numbers using the standard algorithm (**5.NBT.5▲**). Generally the standards distinguish strategies from algorithms. In particular, the "standard algorithm" refers here to multiplying numbers digit-by-digit and recording the products piece-by-piece. Note that the *method of recording* the algorithm is not the same as the *algorithm* itself, in the sense that the "partial products" method, which lists every single digit-by-digit product separately, is a completely valid recording method for the "standard algorithm." Ultimately, the standards call for *understanding* the standard algorithm in terms of place value, and this should be the most important goal for instruction.

[Note: Sidebar]

FLUENCY

In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., "fluently" multiply multi-digit whole numbers using the standard algorithm (**5.NBT.5▲**). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary.

The word "fluent" is used in the standards to mean "reasonably fast and accurate" and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.

In previous grades, students built a conceptual understanding of multiplication with whole numbers as they applied multiple strategies to compute and solve problems. Students can continue to use different strategies and methods from previous years as long as they are efficient, but they must also understand and be able to use the standard algorithm.

Example: Find the product 123 × 34

When students apply the standard algorithm, they decompose 34 into 30 + 4. Then they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place, and add the two products. The ways in which students are taught to record this method may vary, but all should emphasize the place-value nature of the algorithm. For example, one might write

$$\begin{array}{rl} 123 & \\ \times\ 34 & \\ \hline 492 & \leftarrow \text{this is the product of 4 and 123} \\ 3690 & \leftarrow \text{this is the product of 30 and 123} \\ \hline 4182 & \leftarrow \text{this is the sum of the two partial products} \end{array}$$

Note that a further decomposition of 123 into 100 + 20 + 3 and recording of the partial products would also be acceptable.

(Adapted from Arizona 2012).

In grade five students extend division to include quotients of whole numbers with up to four-digit dividends and two-digit divisors using various strategies, and they illustrate and explain calculations by using equations, rectangular arrays, and/or area models. (**5.NBT.6▲**). When the two-digit divisor is a "familiar" number, students might use various strategies based on place value understanding.

Example 1: Find the quotient 2682 ÷ 25

- Using expanded notation: 2682 ÷ 25 = (2000 + 600 + 80 + 2) ÷ 25
- Using an understanding of the relationship between 100 and 25, a student might think:
  - I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80.
  - 600 divided by 25 has to be 24.
  - Since 3 x 25 is 75, I know that 80 divided by 25 is 3 with a reminder of 5. (Note that a student might divide into 82 and not 80)
  - I can't divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
  - 80 + 24 + 3 = 107. So, the answer is 107 with a remainder of 7.

- Using an equation that relates division to multiplication, 25 × *n* = 2682, a student might estimate the answer to be slightly larger than 100 by recognizing that 25 × 100 = 2500.

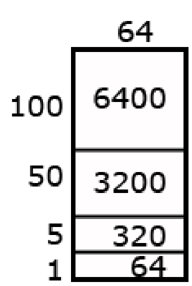
To help students understand the use of place value when dividing with two digit divisors, students can begin with simpler examples, such as dividing150 by 30. Clearly the answer is 5 since this is 15 tens divided by 3 tens. However, when dividing 1500 by 30, students need to think of this as 150 tens divided by 3 tens, which is 50. This illustrates why when using the division algorithm the 5 would go in the tens place of the quotient.

When the divisor is less familiar, students can use strategies based on area such as shown in the following example.

Example 2: Find the quotient 9984 ÷ 64

An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.

Area model:



So the quotient is 100 + 50 + 5 + 1 =156.

Recording:

64	9984	
	−6400	(100 × 64)
	3584	
	−3200	(50 × 64)
	384	
	−320	(5 × 64)
	64	
	−64	(1 × 64)
	0	

(Adapted from Arizona 2012)

The extension from one-digit divisors to two-digit divisors requires care (**5.NBT.6▲**). This is a major milestone along the way to reaching fluency with the standard algorithm in grade six. Division strategies in grade five extend the grade four methods to 2-digit divisors. Students continue to break the dividend into base-ten units and find the quotient place by place, starting from the highest place. They illustrate and explain their calculations using equations, rectangular arrays, and/or area models. Estimating the quotients is a difficult new aspect of dividing by a 2-digit number. Even if students round appropriately, the resulting estimate may need to be adjusted up or down. Students may write any needed new group from multiplying within the division or add it in mentally or write the multiplication out to the side, if necessary.

[Note: Sidebar]

Focus, Coherence, and Rigor:

When students break divisors and dividends into sums of multiples of base-ten units (**5.NBT.6▲**), they also develop important mathematical practices such as how to see and make use of structure (**MP.7**) and attend to precision (**MP.6**). (PARCC 2012).

In grade five students build on work with comparing decimals in fourth grade and begin to add, subtract, multiply, and divide decimals to hundredths (**5.NBT.7▲**). Students focus on reasoning about operations with decimals using concrete models, drawings, various strategies, and explanations. They extend the models and written models they developed for whole numbers in grades one through four to decimal values.

Students might estimate answers based on their understanding of operations and the value of the numbers. **(MP.7, MP.8)**

Examples: Estimate
3.6 + 1.7. A student can make good use of rounding to estimate that since 3.6 rounds up to 4 and 1.7 rounds up to 2, the answer should be close to 4 + 2 = 6.
5.4 − 0.8. Students can again round and argue that since 5.4 rounds down to 5 and 0.8 rounds up to 1, the answer should be close to 5 – 1 = 4.
6 × 2.4. A student might estimate an answer between 12 and 18 since 6 × 2 is 12 and 6 × 3 is 18.

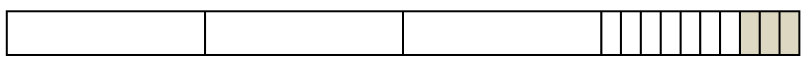
Students must understand and be able to explain that when adding decimals they add tenths to tenths and hundredths to hundredths. When students add in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. Students reinforce their understanding of adding decimals by connecting to prior understanding of adding fractions with denominators of 10 and 100 from fourth grade. Students understand that when adding and subtracting a whole number the decimal point is at the end of the whole number.

Students use various models to support their understanding of decimal operations.

Example 1: (Model for decimal subtraction)

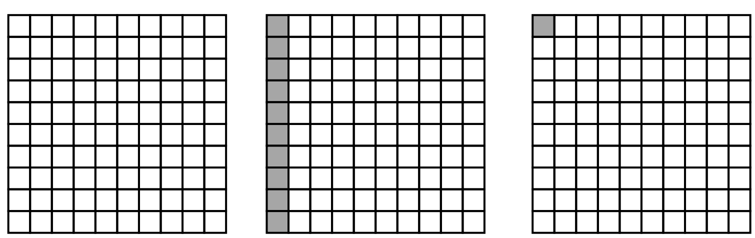
Find $4 - 0.3$. Explain how you found your solution.

"Since I'm subtracting 3 tenths from 4 wholes, it would help to divide one of the wholes into tenths. The other 3 wholes don't need to be divided up. I can see there are 3 wholes and 7 tenths leftover, or 3.7."



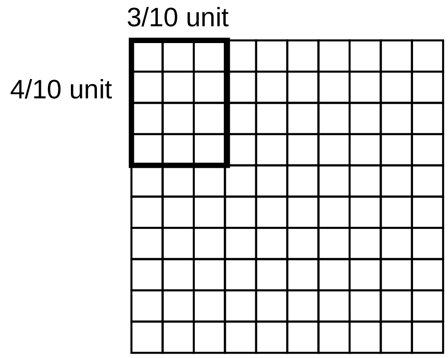
Example 2: Use an area model to demonstrate that $\frac{1}{10}$ of $\frac{1}{10}$ is $\frac{1}{100}$.

"If I use my 10×10 grid and set the whole grid to equal 1 square unit, then I can see that when each length of the grid is divided into ten equal parts, each small square must be representing a $\frac{1}{10}\times\frac{1}{10}$ square. But there are 100 of these small squares in the whole, so each little square must have area $\frac{1}{100}$ square units."



Example 3: Use an area model to demonstrate that $\frac{3}{10}\times\frac{4}{10}=\frac{12}{100}$.

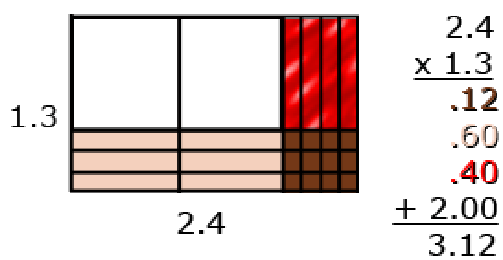
"Just like in the previous problem, I use my 10×10 grid to represent 1 whole, with dimensions 1 unit by 1 unit. If I break up each side length into ten equal parts, then I can create a smaller rectangle of dimensions 3 tenths of a unit by 4 tenths of a unit. It looks something like this:



I know from before that each of the small squares is $\frac{1}{100}$ of a square unit, and I can see there are 3 × 4 = 12 of these small squares in the rectangle I outlined. This shows the answer is $\frac{12}{100}$." (See also **5.NF.4)**

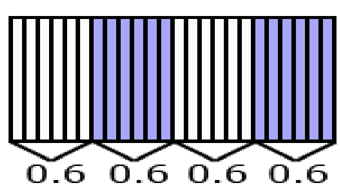
Example 4: Use an area model to show that 2.4 × 1.3 = 3.12.

"I drew a picture that shows a rectangle of lengths 1.3 units and 2.4 units. I know how to break up and keep track of the smaller units like tenths and hundredths. The partial products appear in my picture a lot like the previous problem."



Example 5: (Partitive or "fair-share" division model applied to decimals.) Find 2.4 ÷ 4 and justify your answer.

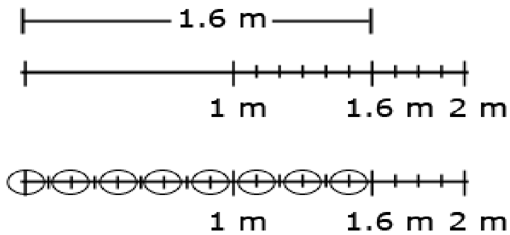
"My partner and I decided to think of this as fair-share division. We drew 2 wholes and 4 tenths, and decided to break the wholes into tenths as well, since it would be easier to share them. When we tried to divide the total number of tenths into four equal parts, we got 0.6 in each part."



Example 6: (Quotitive or "measurement" division model applied to decimals.)

Solve the following problem: "Joe has 1.6 meters of rope. He needs to cut pieces of rope that are 0.2 meters long. How many pieces can he cut?

"We decided to draw a number line segment 2 units long that would represent Joe's 1.6 meters of rope, 1 whole meter and 6 tenths of a meter. Since we need to count smaller ropes that are 0.2 meters in length, we also decided to divide the 1 whole into tenths as well. Then it wasn't too hard to count that there are 8 pieces of 0.2-meter long rope in his 1.6-meter rope."



(Adapted from Arizona 2012 and KATM 5th FlipBook 2012)

Domain: Number and Operations-Fractions

Student proficiency with fractions is essential to success in algebra at later grades. In grade five a critical area of instruction is developing fluency with addition and subtraction of fractions, including adding and subtracting fractions with unlike denominators. Students also build an understanding of multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).

Number and Operations—Fractions 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.*

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases with unlike denominators, e.g., by using visual fraction models or equations to represent the problems. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

In grade four students calculated sums of fractions with different denominators, where one denominator is a divisor of the other, so that only one fraction has to be changed. In grade five students extend work with fractions to add and subtract fractions with unlike

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Perform operations with multi-digit whole numbers and with decimals to hundredths.*

Standard: Grade 5.NBT.5

Fluently multiply multi-digit whole numbers using the standard algorithm**.**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: (5.NBT.5-7)

This cluster is connected to:

- Grade 5 Critical Area of Focus #2, Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.
- Use place value understanding and properties of operations to perform multi-digit arithmetic (Grade 4 NBT 5 and 6).

Explanation and Examples:

This standard refers to fluency which means students select and use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.

- They work flexibly with basic number combinations and use visual models, benchmarks, and equivalent forms.
- They are accurate and efficient (use a reasonable amount of steps), and flexible (use strategies such as the distributive property or breaking numbers apart (decomposing and recomposing) also using strategies according to the numbers in the problem, 26×4 may lend itself to $(25 \times 4) + 4$ where as another problem might lend itself to making an equivalent problem $32 \times 4 = 64 \times 2$).

This standard builds upon students' work with multiplying numbers in third and fourth grade. In fourth grade, students developed understanding of multiplication through using various strategies. While the standard algorithm is mentioned, alternative strategies are also appropriate to help students develop conceptual understanding.

The size of the numbers should **NOT** exceed a three-digit factor by a two-digit factor.

In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand and be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value.

Example:

Find the product of 123×34. When students apply the standard algorithm, they, decompose 34 into $30 + 4$. Then they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place, and add the two products. The ways in which students are taught to record this method may vary, but ALL should emphasize the place-value nature of the algorithm, For example, one might write

123		
×34		
492	←	this is the product of 4 and 123
3690	←	this is the product of 30 and 123
4182	←	this is the produce of the two partial products

Note that a further decomposition of 123 into 100 + 20 + 30 and recording of the partial products would also be acceptable.

Examples of alternative strategies:

There are 225 dozen cookies in the bakery. How many cookies are there?

Student 1

- 225×12
- *I broke 12 up into 10 and 2.*
- $225 \times 10 = 2{,}250$
- $225 \times 2 = 450$
- $2{,}250 + 450 = 2{,}700$

Student 2

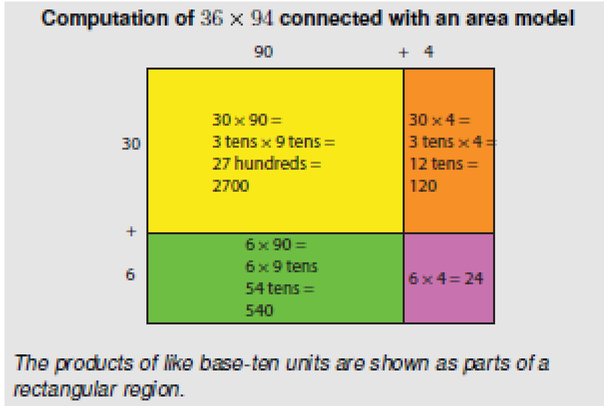
- 225×12
- *I broke up 225 into 200 and 25.*
- $200 \times 12 = 2{,}400$
- *I broke 25 up into* 5×5, *so I had* $5 \times 5 \times 12$ *or* $5 \times 12 \times 5$.
- $5 \times 12 = 60, 60 \times 5 = 300$
- *I then added 2,400 and 300* $2{,}400 + 300 = 2{,}700$

Student 3

- I doubled 225 and cut
- 12 in half to get 450×6.
- I then doubled 450
- again and cut 6 in half
- to get 900×3.
- $900 \times 3 = 2{,}700$

Draw an array model for

$$36 \times 94 = (30 + 6) \times (90 + 4) = (30 + 6) \times 90 + (30 + 6) \times 4 = 30 \times 90 + 6 \times 90 + 30 \times 4 + 6 \times 4$$



Taken from Progression for the Common Core: K-5, Number and Operations in Base Ten
(Click array to open complete document.)

Instructional Strategies: (5.NBT.5-7)

Because students have used various models and strategies to solve problems involving multiplication with whole numbers, they should be able to transition to using standard algorithms effectively. With guidance from the teacher, they should understand the connection between the standard algorithm and their strategies.

Connections between the algorithm for multiplying multi-digit whole numbers and strategies such as partial products or lattice multiplication are **necessary for students' understanding.**

You can multiply by listing all the partial products. For example:

234	
× 8	
32	Multiply by the ones ($8 \times 4\ ones = 32\ ones$)
240	Multiply by the tens ($8 \times 3\ tens = 24\ tens\ or\ 240$)
1,600	Multiply the hundreds ($8 \times 2\ hundreds = 16\ hundreds\ or\ 1{,}600$)
1,872	Add the partial products

The multiplication can also be done without listing the partial products by multiplying the value of each digit from one factor by the value of each digit from the other factor. Understanding of place value is vital in using the standard algorithm.

In using the standard algorithm for multiplication, when multiplying the ones, 32 ones is 3 tens and 2 ones. The 2 is written in the ones place. When multiplying the tens, the 24 tens is 2 hundreds and 4 tens. But, the 3 tens from the 32 ones need to be added to these 4 tens, for 7 tens. Multiplying the hundreds, the 16 hundreds is 1 thousand and 6 hundreds. But, the 2 hundreds from the 24 tens need to be added to these 6 hundreds, for 8 hundreds.

234
× 8
1,872

As students developed efficient strategies to do whole number operations, they should also develop efficient strategies with decimal operations.

Students should learn to **estimate** decimal computations **before** they compute with pencil and paper. The focus on estimation should be on the meaning of the numbers and the operations, not on how many decimal places are involved.

For example, to estimate the product of 32.84×4.6, the estimate would be more than 120, closer to 150. Students should consider that 32.84 is closer to 30 and 4.6 is closer to 5. The product of 30 and 5 is 150. Therefore, the product of 32.84×4.6 should be close to 150. *(Writing equations horizontally encourages using mental math).*

Have students use estimation to find the product by using exactly the same digits in one of the factors with the decimal point in a different position each time. For example, have students estimate the product of 275×3.8; 27.5×3.8 *and* 2.75×3.8, and discuss **why** the estimates should or should not be the same.

Common Misconceptions: (5.NBT.5-7)

Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number. For example, in computing the sum of 15.34 + 12.9, students will write the problem in this manner:

$$\begin{array}{r} 15.34 \\ +\ \ 12.9 \\ \hline 16.63 \end{array}$$

To help students add and subtract decimals correctly, have them **first estimate** the sum or difference. Providing students with a decimal-place value chart will enable them to place the digits in the proper place.

Domain: Number and Operations in Base Ten (NBT)

Cluster: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Standard: Grade 5. NBT.6

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 5.NBT.5

Explanation and Examples:

This standard references various strategies for division. Division problems can include remainders. Even though this standard leads more towards computation, the connection to story contexts is **critical**.

Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups. In fourth grade, students' experiences with division were limited to dividing by one-digit divisors.

This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Example:
There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created? If you have left over students, what do you do with them?

<u>Student 1</u>

1,716 divided by 16
There are 100 16's in 1,716.

$$1{,}716 - 1{,}600 = 116$$

I know there are at least 6 16's.

$$116 - 96 = 20$$

I can take out at least 1 more 16.

$$20 - 16 = 4$$

There were 107 teams with 4 students left over. If we put the extra students on different team, 4 teams will have 17students.

<u>Student 2</u>

1,716 divided by 16
There are 100 16's in 1,716.
Ten groups of 16 is 160.
That's too big.
Half of that is 80, which is5 groups.
I know that 2 groups of 16's is 32.
I have 4 students left over.

$1{,}716 - 1{,}600$	100
$116 - 80$	5
36-32	2
4	

<u>Student 3</u>

$$1{,}716 \div 16 = ?$$

I want to get to 1,716
I know that 100 16's equals 1,600
I know that 5 16's equals 80

$$1{,}600 + 80 = 1{,}680$$

Two more groups of 16's equals 32, which gets us to 1,712.
I am 4 away from 1,716.
So we had $100 + 6 + 1 = 107$ teams.
Those other 4 students can just hang out.

<u>Student 4</u>

How many 16's are in 1,716?
We have an area of 1,716.
I know that one side of my array is 16 units long.
I used 16 as the height.
I am trying to answer the question what is the width of my rectangle if the area is 1,716 and the height is 16.

$$100 + 7 = 107\ R4$$

	100	7
16	$100 \times 16 = 1{,}600$	$7 \times 16 = 112$

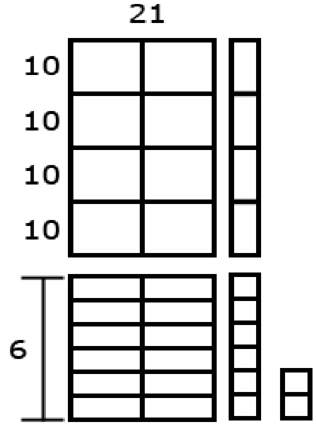
$$1{,}716 - 1{,}600 = 116$$
$$116 - 112 = 4$$

Major Supporting Additional **Depth Opportunities(DO)**

Example:

$968 \div 21 = ?$

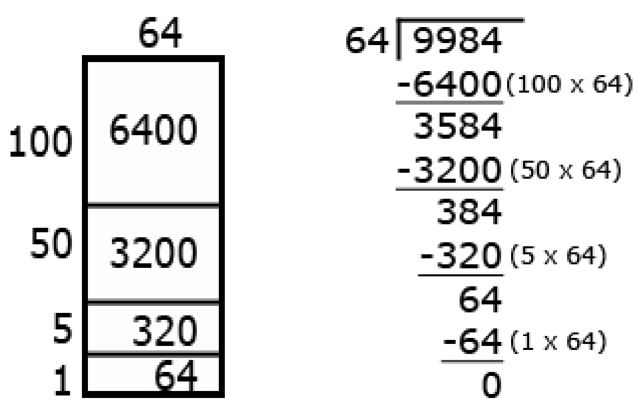
Using base ten models, a student can represent 968 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.



Example:

$9{,}984 \div 64 = ?$

An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.



Instructional Strategies: See Grade 5. NBT. 5

Resources/Tools:

- With this activity, you can visually explore the concept of factors by creating rectangular arrays. The length and width of the array are the factors in your number. a[http://illuminations.nctm.org/ActivityDetail.aspx?ID=64
- Learning Progressions for Numbers and Operations in Base Ten

Common Misconceptions: See Grade 5. NBT. 5

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Perform operations with multi-digit whole numbers and with decimals to the hundredths.*

Standard: Grade 5. NBT.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 5.NBT.5

Explanation and Examples:

This standard builds on the work from fourth grade where students are introduced to decimals and compare them. In fifth grade, students begin adding, subtracting, multiplying and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations ($2.25 \times 3 = 6.75$), but this work should not be done without models or pictures.

This standard includes students' reasoning and explanations of how they use models, pictures, and strategies. This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.

Examples:

$$3.6 + 1.7 = ?$$

A student might estimate the sum to be larger than 5 because 3.6 is more than $3\frac{1}{2}$ and 1.7 is more than $1\frac{1}{2}$.

$$5.4 - 0.8 = ?$$

A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.

$$6 \times 2.4 = ?$$

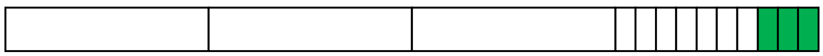
A student might estimate an answer between 12 and 18 since 6×2 is 12 and 6×3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than $6 \times 2\frac{1}{2}$ and think of $2\frac{1}{2}$ groups of 6 as 12 (2 groups of 6) + 3 ($\frac{1}{2}$ of a group of 6).

Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.

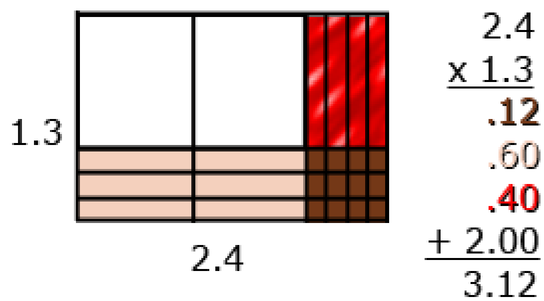
Example:

$$4 - 0.3 = ?$$

3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.



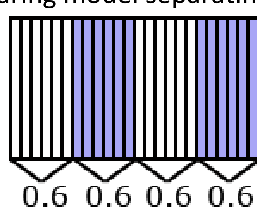
The answer is 3 and $\frac{7}{10}$ or 3.7.



Students should be able to **describe** the partial products displayed by the area model. For example, "$\frac{3}{10}$ times $\frac{4}{10}$ is $\frac{12}{100}$. $\frac{3}{10}$ times 2 is $\frac{6}{10}$ or $\frac{60}{100}$. 1 group of $\frac{4}{10}$ is $\frac{4}{10}$ or $\frac{40}{100}$. 1 group of 2 is 2."

Example of division: finding the number in each group or share

Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as:

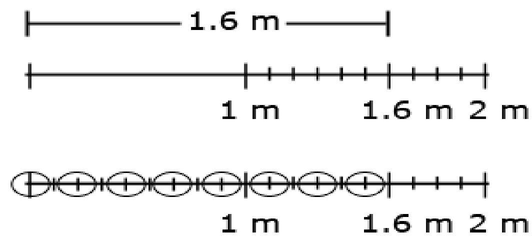


Example of division: find the number of groups

- Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?

- To divide to find the number of groups, a student might:
  - Draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.

Major Supporting Additional **Depth Opportunities(DO)**



- Count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as $\frac{10}{10}$, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . . 16 tenths, a student can count 8 groups of 2 tenths.
- Use their understanding of multiplication and think, "8 groups of 2 is 16, so 8 groups of $\frac{2}{10}$ is $\frac{16}{10}$ or $1\frac{6}{10}$."

Instructional Strategies: **See Grade 5.NBT.5**

Common Misconceptions: See Grade 5. NBT. 5

SCUSD 5th Grade Curriculum Map

Unit 2: Multi-digit Operations with Whole Numbers an Decimal Fractions
Sequence of Learning Outcomes **5.NBT.5, 5.NBT.6, 5.NBT.7**
1) Multiply multi-digit whole numbers using mental strategies.
2) Apply place value knowledge to fluently multiply multi-digit numbers using a partial products method and to solve multi-step, real world word problems (add, and/or subtract, and multiply).
3) Divide multi-digit whole numbers using mental strategies, including manipulatives & estimation.
4) Apply place value knowledge to extend multi-digit division of numbers up to four-digit dividends and two-digit divisors to solve multi-step, real world word problems. Students will use various strategies, such as the partial quotient method, area model, drawings, and equations to illustrate and explain their reasoning.
5) Add and subtract decimals up to hundredths (using a variety of strategies) and apply to word problems.
6) Apply place value knowledge to extend to decimal multi-digit multiplication (up to hundredths) using concrete models, drawings, various strategies, and explanations.
7) Extend division understanding from whole numbers to decimal numbers (up to hundredths) using concrete models, drawings, various strategies, and written models.

envision Learning Outcomes
5.NBT.5, 5.NBT.6, 5.NBT.7

Topic 2: Adding and Subtracting Decimals
Sequence of learning objectives **Lesson 2.1, 2.3 – 2.7**
2.1 Mental Math • Students compute sums and differences mentally using the Commutative and Associative Properties of Addition, compatible numbers, and compensation.
2.3 Estimating Sums and Differences • Students use rounding and compatible numbers to estimate sums and differences of whole numbers and decimals.
2.4 Modeling Addition and Subtracting of Decimals • Students will add and subtract decimals in tenths and hundredths using models.
2.5 Adding Decimals • Students compute sums of decimals involving tenths and hundredths.
2.6 Subtracting Decimals • Students compute differences of decimals involving tenths and hundredths.
2.7 Problem Solving: Multiple Step Problems • Students use multiple steps to solve a variety of problems.
Topic 3: Multiplying Whole Numbers
Sequence of Learning Objectives **Lessons 3.1, 3.3 – 3.6**
3.1 Multiplication Properties • Students identify and apply the Commutative, Associative, Identity, and Zero Properties of Multiplication. • This will prepare students to apply these properties when multiplying decimals.
3.3 Multiplying 2-Digit Numbers by Multiples of 10 • Students will use the standard algorithm and area models to multiply 2-digit number by multiples of ten.
3.4 Multiplying 2-digit by 2-Digit Numbers • Students multiply two-digit numbers by two-digit numbers.
3.5 Multiplying Greater Numbers • Students multiply two-digit numbers by factors with more than two digits.
3.6 Problem Solving: Draw a Picture and Write an Equation • Students use diagrams and write equations to solve problems.
Topic 4: Dividing by 1-Digit Divisors
Sequence of Learning Objectives **Lessons 4.1 – 4.7**
4.1 Dividing Multiples of 10 and 100 • Students find the quotient of a division problem whose dividend is a multiple of 10, where division involves a basic fact.
4.2 Estimating Quotients • Students use rounding and compatible numbers to estimate quotients of whole numbers.
4.3 Problem Solving: Reasonableness • Students check problems for reasonableness by using various methods, including estimation and checking their final answer.
4.4 Dividing by 1-Digit Divisors • Students divide three-digit and four-digit whole numbers by one-digit divisors.
4.5 Zeros in the Quotient • Students divide with zeros in the quotient.
4.6 More Dividing by 1-Digit Divisors

• Students will divide 3-digit and 4-digit whole numbers by 1-digit divisors and use estimation to check quotients for reasonableness.
4.7 Problem Solving : Draw a Picture and Write an Equation • Students use pictures and equations to help them represent remainders in a problem.
Topic 5: Dividing by 2-Digit Divisors
Sequence of Learning Objectives **Lessons 5.1 – 5.8**
5.1 Using Patterns to Divide • Students find the quotients of division problems whose dividends and divisors are multiples of 10, where the division involves a basic fact.
5.2 Estimating Quotients with 2-Digit Divisors • Students use estimation to find approximate solutions to division problems with two-digit divisors using compatible numbers.
5.3 Connecting Models and Symbols • Students will use arrays and area models to model division.
5.4 Dividing by Multiples of 10 • Students find quotients with a two-digit divisor that is a multiple of ten.
5.5 1-Digit Quotients • Students find one-digit quotients where the divisor is a two-digit number.
5.6 2-Digit Quotients • Students divide a three-digit number to find a two-digit quotient.
5.7 Dividing with Greater Numbers • Students solve problems involving division of numbers with 4 or 5 digits by 2-digit divisors with an estimate, or by using a calculator when the exact answer is needed.
5.8 Problem Solving: Missing or Extra Information • Students determine which information is missing and identify extraneous information in problems.
Topic 6: Multiplying Decimals
Sequence of Learning Objectives **Lessons 6.2 – 6.7**
6.2 Estimating the Product of Decimal and a Whole Number • Students use rounding and compatible numbers to estimate products of whole numbers and decimals. • Students also identify estimates as overestimates or underestimates.
6.3 Number Sense: Decimal Multiplication • Students will use number sense and place value to multiply decimals.
6.4 Models for Multiplying Decimals • Students find products of whole numbers and decimals to ten thousandths.
6.5 Multiplying a Decimal by a Whole Number • Students use a standard algorithm to multiply a whole number and a decimal.
6.6 Multiplying Two Decimals • Students will use the standard algorithm to multiply decimals by decimals.
6.7 Problem Solving: Multiple-Step Problems • Students find the hidden question or questions to solve multiple-step problems.
Topic 7: Dividing Decimals
Sequence of Learning Objectives **Lessons 7.2 – 7.7**
7.2 Estimating Decimal Quotients • Students will learn to estimate quotients involving decimals, and to use reasoning to understand how the size of the quotient relates to the dividend and divisor.
7.3 Number Sense: Decimal Division

• Students will learn how to use reasoning to correctly place the decimal point in a quotient.
7.4 Dividing by a Whole Number • Students find quotients where the dividend and/or the quotient is a decimal.
7.5 Dividing a Whole Number by a Decimal • Students divide whole numbers by decimals.
7.6 Dividing a Decimal by a Decimal • Students find quotients of two decimals.
Problem Solving: Multiplying-Step Problems • Students use multiple steps to solve a variety of problems.

Name ____________________________________

Mark the best answer.

1. Melissa earned $296 by raking leaves for 5 weeks. Which number sentence shows the best way to estimate the amount she earned each week? (4-2)

A $5 \times \$300 = \$1,500$

B $\$300 \div 10 = \$30$

C $\$250 \div 5 = \$50$

D $\$300 \div 5 = \$60$

2. If $900 is divided evenly by 9 people, how many dollars does each person get? (4-1)

A $10

B $100

C $1,000

D $10,000

3. If 327 is divided by 6, where should the first digit of the quotient be placed? (4-4)

A Because 6 is greater than 3, it should be placed in the tens place.

B Because 6 is less than 3, it should be placed in the tens place.

C Because 6 is greater than 3, it should be placed in the hundreds place.

D Because 6 is less than 3, it should be placed in the hundreds place.

4. What is the most reasonable estimate of $155 \div 9$? (4-3)

A 7

B 9

C 12

D 16

5. The table shows the boxes of cookies sold by students as a school fundraiser. Marc sold 3 times the number that Karen did. How many boxes of cookies did Karen sell? (4-5)

Student	Boxes
Marc	612
Karen	?
Mia	97

A 204

B 173

C 136

D 121

Name ______________________________

6. Which of the following shows the best way to estimate $728 \div 8$ using compatible numbers? (4-2)

   A $700 \div 8$

   B $720 \div 8$

   C $740 \div 8$

   D $800 \div 8$

7. A summer camp has 110 campers divided as evenly as possible into 9 cabins. About how many campers are in each cabin? (4-4)

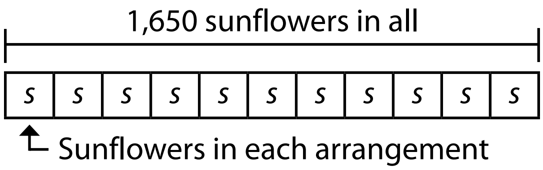
8. A triathlon race has 416 athletes divided as evenly as possible into 6 groups. About how many athletes are in each group? Use compatible numbers to solve. (4-3)

9. A group of 8 students is sharing 320 comic books equally. How many comic books does each student get? (4-1)

10. A team of 5 bowlers is sharing a prize of $1525 equally. How much money does each bowler get? (4-5)

Name ______________________________

11. A florist is making flower arrangements. He has 1,650 sunflowers to place in 11 arrangements. What equation can be used to find *s*, the number of sunflowers in each arrangement? (4-7)



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12. Mrs. Kay wants to buy as many packages of colored pencils as possible. Each package costs \$3, and she has \$72. How many packages can she buy? (4-4)

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13. Write an equation to show the best way to estimate $264 \div 9$ using compatible numbers. (4-2)

____

____

14. For each pair, determine if the quotient is the same or different. Mark *Yes* or *No*. (4-1)

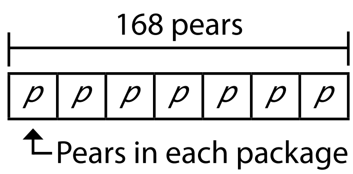
$1{,}200 \div 3$ and $4{,}000 \div 10$
A Yes **B** No

$6{,}400 \div 8$ and $3{,}200 \div 4$
A Yes **B** No

$63{,}000 \div 9$ and $48{,}000 \div 6$
A Yes **B** No

$56{,}000 \div 7$ and $16{,}000 \div 2$
A Yes **B** No

15. Ollie's Orchard has 168 pears ready to ship. If 24 pears fit in each package, how many packages can be filled? (4-7)



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Illustrative Mathematics

5.NBT.B.5 Elmer's Multiplication Error

This is Elmer’s work on a multiplication problem:

$$\begin{array}{r} {\scriptstyle 4\;5\;\;} \\ {\scriptstyle \cancel{3}\;\cancel{3}\;\;} \\ 179 \\ \times\ 64 \\ \hline 716 \\ +\quad 1{,}074 \\ \hline 1{,}790 \end{array}$$

a. Use estimation to explain why Elmer’s answer is not reasonable.

b. What error do you think Elmer made? Why do you think he made that error?

c. Find 179 x 64 using a correct version of Elmer's method. Then show another way of doing it to help Elmer see why your answer is correct.

5.NBT.B.7 The Value of Education

The table shows four people who earn the typical amount for their education level.

Name	Level of Education	Weekly Income
Miley	High School Dropout	$440.50
Niko	High School Graduate	$650.35
Taylor	2-Year College Graduate	$771.25
Pinky	4-Year College Graduate	$1,099.20

a. How much more does Niko earn than Miley in one week?

b. If Taylor and Miley both work for 2 weeks, how much more will Taylor earn?

c. How much money will Pinky earn in a month? About how long will Miley have to work to earn the same amount?