CCSS-M Teacher Professional Learning

Session #1, October 2014

Grade 4

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(Selected pages relevant to session work)

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Grade 4

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten[2] 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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[1] See Glossary, Table 2.

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions[3] 4.NF

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.
    a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
    b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
    c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
    d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
    a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
    b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
    c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Common Core State Standards - Mathematics
Standards for Mathematical Practices – 4th Grade

Standard for Mathematical Practice	4th Grade
1: Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, -Does this make sense? They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

2: Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3: Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like -How did you get that? and -Why is that true? They explain their thinking to others and respond to others' thinking.

4: Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
5: Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose	Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.

6: Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7: Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.

8: Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Students need multiple opportunities to use real-world contexts to read and write multi-digit whole numbers. Student need to reason about the magnitude of digits in a number and analyze the relationships of number. They can build larger numbers by using graph paper with very small squares and labeling examples of each place with digits and words (e.g., ten thousand and 10,000).

To read and write numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (e.g., thousand, million). Layered place value cards such as those used in earlier grades can be put on a frame with the base-thousand units labeled below. Then cards forming hundreds, tens, and ones can be placed on each section and the name read off using the card values followed by the word “million”, then “thousand”, then the silent ones **(MP.2, MP.3, MP.8).**

Fourth-grade students build on the grade-three skill of rounding to the nearest 10 or 100 to round multi-digit numbers and to make reasonable estimates of numerical values. (**4.NBT.3▲**).

Example: Rounding Numbers in Context. (MP.4)

The population of Midtown, U.S.A., was last recorded to be 76,398. The city council wants to round the population to the nearest thousand for a business brochure. What number should they round the population to?

Solution: When students represent numbers stacked vertically, they can see the relationships between the numbers more clearly. Students might think: “I know the answer is either 76,000 or 77,000. If I write 76,000 below 76,398 and 77,000 above it, I can see that the midpoint is 76,500, which is *above* 76,398. This tells me they should round the population to 76,000.”

77,000
76,398
76,000

Numbers and Operations in Base Ten 4.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-

digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

At grade four, students become fluent with addition and subtraction with multi-digit whole numbers to 1,000,000 using standard algorithms (**4.NBT.4▲**). A central theme in multi-digit arithmetic is to encourage students to develop methods they understand, can explain, and can think about, rather than merely following a sequence of directions, rules or procedures they do not understand. In previous grades, students built a conceptual understanding of addition and subtraction with whole numbers as they applied multiple methods to compute and solve problems. The emphasis in grade four is on the power of the regular one-for-ten trades between adjacent places that let students extend a method they already know to many places. Because students in grades two and three have been using at least one method that will generalize to 1,000,000, this extension in grade four should not have to take a long time. Thus, students will also have sufficient time for the major new topics of multiplication and division (**4.NBT.5-6▲**).

[Note: Sidebar]

Fluency
In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., “fluently” add and subtract multi-digit whole numbers using the standard algorithm (**4.NBT.4▲**)). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary.
The word “fluent” is used in the standards to mean “reasonably fast and accurate” and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies (Adapted from Progressions K-5 CC and OA 2011 and PARCC 2012).

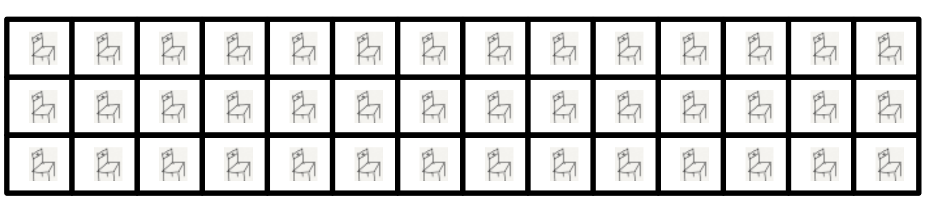
In grade four students extend multiplication and division to include whole numbers greater than 100. Students should use methods they understand and can explain to multiply and divide. The standards (**4.NBT.5-6▲**) call for students to use visual representations such as area and array models that students draw and connect to equations and written numerical work that supports student reasoning and explanation of methods. By reasoning repeatedly about the connections between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities.

After students have discussed how to show an equal groups situation or a multiplication compare situation with an area model, they can use area models for any multiplication situation. The rows represent the equal groups of objects or the larger compared quantity and students imagine that the objects in the situation lie in the squares and so form an array. Such array models become too difficult to draw, so students can make sketches of rectangles and then label the resulting product as the number of things or square units. When using area models to represent an actual area situation, the two factors are in length units (e.g., cm) while the product is in square units (e.g., cm^2).

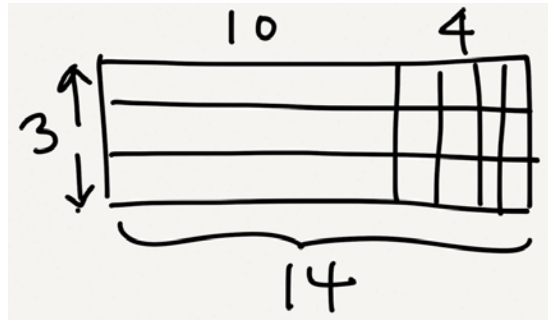
Example: Area Models and Strategies for Multi-digit Multiplication , Single Digit Multiplier (4.NBT.5▲)

"Chairs are being set up for a small play. There should be 3 rows of chairs and 14 chairs in each row. How many chairs will be needed?"

Solution: As in grade three, when students first made the connection between array models and the area model, students might start by drawing a sketch of the situation. They can then be reminded to see the chairs as if surrounded by unit squares and hence a model of a rectangular region. With base-ten blocks or math drawings **(MP.2, MP.5)**, students abstract the problem and see it being broken down into $3 \times (10 + 4)$.



Making a sketch like the one above becomes cumbersome, so students move toward representing such drawings abstractly, with rectangles, as shown to the right. This builds on the work begun in grade 3. Such diagrams help children see the distributive property: "3×14 can be written as $3 \times (10 + 4)$, and I can do the multiplications separately and add the results, $3 \times (10 + 4) = 3 \times 10 + 3 \times 4$. The answer is $30 + 12 = 42$, or 42 chairs."



In grade three students worked with multiplying single digit numbers by multiples of 10 **(3.NBT.3)**. This idea is extended in grade four, e.g., since $6 \times 7 = 42$, it must be true that:

- $6 \times 70 = 420$, since this is "six times seven tens," which is 42 tens,
- $6 \times 700 = 4200$, since this is "six times seven hundreds," which is 42 hundreds,
- $6 \times 7000 = 42{,}000$, since this is "six times seven thousands," which is 42 thousands,
- $60 \times 70 = 4200$, since this is "sixty times seven tens," which is 420 tens, or 4200.

Math drawings and base-ten blocks support the development of these *extended multiplication facts*. The ability to find products such as these is important when using variations of the standard algorithm for multi-digit multiplication, described below.

Examples: Developing Written Methods for Multi-Digit Multiplication. (4.NBT.5▲)

Left to right Right to left Right to left

Find the product: 6×729.

Solution:

Showing the partial products:

	729	
×	6	t□inking:
	4200	6 × 7 hundreds
	120	6 × 2□tens
	54	6 × 9
	4374	

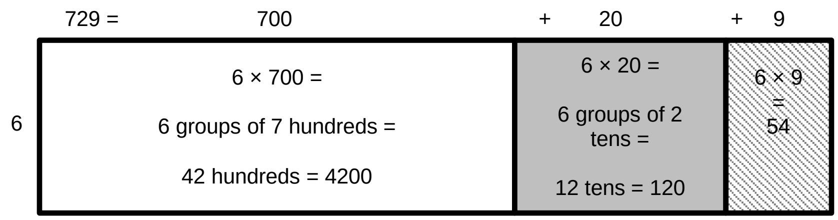
Showing the partial products:

729	
× 6	
54	6 × 9
120	6 × 2 tens
4200	6 × 7 hundreds
4374	

Recording the "carries" below:

729
× 6
1 5
4224
4374

Sufficient practice with drawing rectangles (or constructing them with base-ten blocks) will help students understand that the problem can be represented with a rectangle such as the one shown.



The product is given by the total area: $6 \times 729 = 6 \times 700 + 6 \times 20 + 6 \times 9$. Understanding extended multiplication facts allows students to find the *partial products* quickly. Student can record the multiplication in several ways:

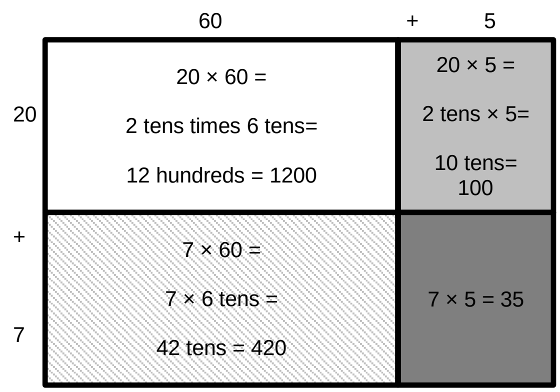
Find the product: 27×65.

Solution: This time, a rectangle is drawn, and "like" base-ten units (e.g., tens and ones) are represented by sub-regions of the rectangle. Repeated use of the distributive property shows that:

$$27 \times 65 = (20 + 7) \times 65 = 20 \times 65 + 7 \times 65$$

$$= 20 \times (60 + 5) + 7 \times (60 + 5)$$

$$= 20 \times 60 + 20 \times 5 + 7 \times 60 + 7 \times 5.$$



The product is again given by the total area:

$$1200 + 100 + 420 + 35 = 1755.$$

Below are two written methods for recording the steps of the multiplication.

Showing the partial products:

65	
× 27	thinking:
35	7 × 5
420	7 × 6 tens
100	2 tens × 5
1200	2 tens × 6 tens
1755	

Recording the carries below for correct place value placement:

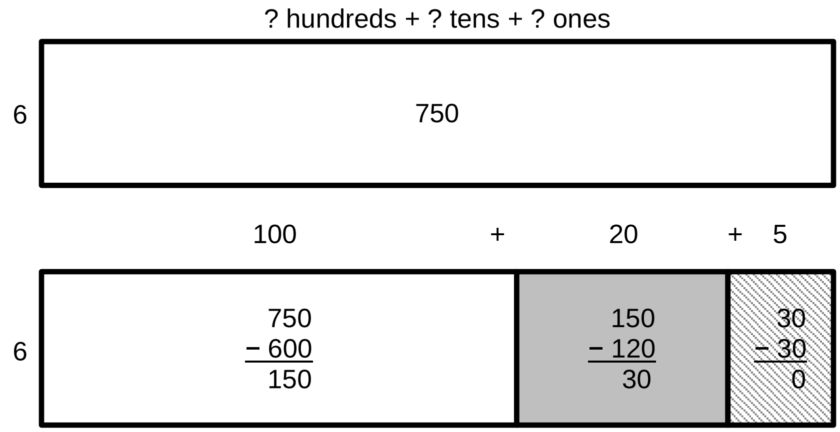
65
× 27
4 3
25
1 1
20**0**
1755

Notice that the boldfaced **0** is included in the second method, indicating that we are multiplying not just by 2 in this row, but by 2 *tens*.

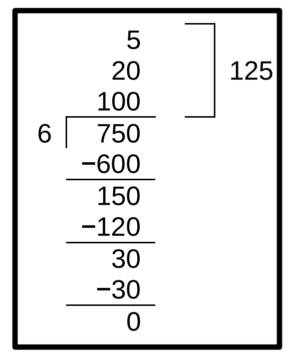
General methods for computing quotients of multi-digit numbers and one-digit numbers **(4.NBT.6▲)** rely on the same understandings as for multiplication, but these are cast in terms division. For example, students may see division problems as knowing the area of a rectangle but not one side length (the quotient) or as finding the size of a group when the number of groups is known (measurement division).

Example: Using the Area Model to Develop Division Strategies.

Find the quotient: 750 ÷ 6.



Solution: "Just like with multiplication, I can set this up as a rectangle, but with one side unknown since this is the same as $?? \times 6 = 750$. I find out what the number of hundreds would be for the unknown side length; that's 1 hundred or 100, since 100 × 6 = 600 and that's as large as I can go. Then, I have $750 - 600 = 150$ square units left, so I find the number of tens that are in the other side. That's 2 tens or 20, since 20 × 6 = 120. Last, there are $150 - 120 = 30$ square units left, so the number of ones on the other side must be 5 since $5 \times 6 = 30$."



One way students can record this is shown, wherein *partial quotients* are stacked atop one another, with 0s included to indicate place value and as a reminder of how students obtained the numbers. The full quotient is the sum of these stacked numbers.

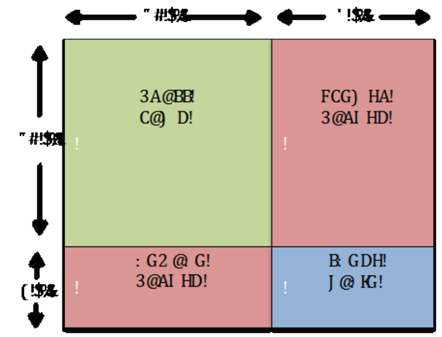
General methods for multi-digit division computation include decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this relies on the distributive property. This work will continue in grade five and culminate in fluency with the standard algorithm in grade six (Adapted from PARCC 2012).

In grade four students also find whole number quotients with remainders **(4.NBT.6▲).** When students experience finding remainders, they should learn the appropriate way to write the result. For instance, students divide and find that $195 \div 9 = 21$ with 6 leftover. This can be written as $195 = 21(9) + 6$. When put into a context, the latter equation makes sense. For instance, if 195 books are distributed equally among 9 classrooms, then each classroom gets 21 books with 6 books leftover. The equation $195 = 21(9) +$

6 is closely related to the equation $195 \div 9 = 21\frac{6}{9}$ which students will write in later grades. The notation $195 \div 9 = 21 \text{ R } 6$ is best avoided.

As students decompose numbers to solve multiplication problems they also reinforce important mathematical practices such as seeing and making use of structure (**MP.7**). As they illustrate and explain calculations they model (**MP.4**), use appropriate drawings as tools strategically (**MP.5**) and attend to precision (**MP.6**) using base-ten units.

Following is a sample problem that connects the Standards for Mathematical Content and the Standards for Mathematical Practice.

Standards	***Explanations and Examples***
4.NBT.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and properties of operations. Illustrate and explain the calculation using equations, rectangular arrays, and/or area models. 4.MD.3: Apply the area and perimeter formals for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	**Sample Problem:** What are the areas of the four sections of Mr. Griffin's backyard? There is a grass lawn, a flower garden, a tomato garden, and a stone patio. What is the area of his entire backyard? How did you find your answer?  18 × 14 Area of Stone Patio→ 32 (4×8) Area of Tomato Garden→ 40 (4×10) Area of Flower Garden→ 80 (10×8) Area of Grass Lawn→100 (10×10) Area of Entire Backyard→ 252 (14×18) **Solution:** The areas of the four sections are 100 sq. ft., 80 sq. ft., 40 sq. ft., and 32 sq. ft. respectively. The area of the entire backyard is the sum of these areas, (100+80+40+32) sq. ft., or 252 sq. ft. This is the same as finding the product (18×14) sq. ft. **Classroom Connections:** The purpose of this task is to illuminate the connection between the area of a rectangle as representing the product of two numbers and the partial products algorithm for multiplying multi-digit numbers. In this algorithm, which is shown to the right, each digit of one number is multiplied by the each digit of the other number and the "partial products" are written down. The sum of these partial products is the product of the original numbers. Place value can be emphasized by specifically reminding students that if we multiply the two 10s together, since each represents one 10, their product is 100. Finally, the area model provides a visual justification for how the algorithm works. **Connecting to the Standards for Mathematical Practice:** (MP.1) Students make sense of the problem when they see that the measurements on the side and top of the diagram persist and yield the measurements of the smaller areas. (MP.2) Students reason abstractly as they represent the areas of the yard as multiplication problems to be solved. (MP.5) Students use appropriate tools strategically when they apply the formula for the area of a rectangle to solve the problem. They organize their work in a way that makes sense to them. (MP.7) Teachers can use this problem and similar problems to illustrate the distributive property of multiplication. In this case, we have that 18×14 = (10×14) + (8×14) = (10×10) + (10×4) + (8×10) + (8×4).

Domain: Number and Operations in Base Ten (NBT)

***Cluster**: Use place value understanding to perform multi-digit arithmetic.*

Standard: Grade 4.NBT.4

Fluently add and subtract multi-digit whole numbers using the standard algorithm**.**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: (4.NBT.4-6)

This Cluster is connected to:

- Fourth Grade Critical Areas of Focus #1 , Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends, and go beyond to address adding and subtracting multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 3 NBT 2 – 3)
- Use the four operations with whole numbers to solve problems (Grade 4 OA 2 – 3).
- Generalize place value understanding for multi-digit whole numbers (Grade 4 NBT 1 – 2).

Explanation and Examples:

Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.

This standard refers to **fluency, which means accuracy and efficiency (using a reasonable amount of steps and time), and flexibility (using a variety of strategies such as the distributive property, decomposing and recomposing numbers, etc.).**

Kansas State Department of Education White Paper on Fluency

This is the first-grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use.

When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.

$$\begin{array}{r} 3892 \\ \underline{+1567} \end{array}$$

Student explanation for this problem:

1. Two ones plus seven ones is nine ones.
2. Nine tens plus six tens is 15 tens.
3. I am going to write down five tens and think of the 10 tens as one more hundred.(notates with a 1 above the hundreds column)
4. Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.
5. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (notates with a 1 above the thousands column)
6. Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.

$$\begin{array}{r} 3546 \\ \underline{-\ 928} \end{array}$$

Student explanation for this problem:

1. There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.)
2. Sixteen ones minus 8 ones is 8 ones. (Writes an 8 in the ones column of answer.)
3. Three tens minus 2 tens is one ten. (Writes a 1 in the tens column of answer.)
4. There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (Marks through the 3 and notates with a 2 above it.) (Writes down a 1 above the hundreds column.) Now I have 2 thousand and 15 hundreds.
5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (Writes a 6 in the hundreds column of the answer).
6. I have 2 thousands left since I did not have to take away any thousands. (Writes 2 in the thousands place of answer.)

Note: Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.

Instructional Strategies: **(4.NBT.4-6)**

A crucial theme in multi-digit arithmetic is encouraging students to develop *strategies* that they understand, can explain, and can think about, rather than merely follow a sequence of directions, rules or procedures that they don't understand. It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to *understand* all the steps in the algorithm, and they should be able to explain the meaning of each digit.

For example, a 1 can represent one, ten, or hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately.

Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.

Sometimes students benefit from 'being the teacher' to an imaginary student who is having difficulties applying standard algorithms in addition and subtraction situations. To promote understanding, use examples of student work that have been done incorrectly and ask students to provide feedback about the student work.

It is very important for some students to talk through their understanding of connections between different strategies and standard addition and subtractions algorithms. Give students many opportunities to talk with classmates about how they could explain standard algorithms. Think-Pair-Share is a good protocol for all students.

When asking students to gain understanding about multiplying larger numbers be sure to provide frequent opportunities to engage in mental math exercises. When doing mental math, it is difficult to even *attempt* to use a strategy that one does not fully understand. Also, it is a natural tendency to use numbers that are 'friendly' (multiples of 10) when doing mental math, and this promotes its understanding.

Tools/Resources

See: "Grocery Shopping, Georgia Department of Education. This task provides students with the opportunity to apply estimation strategies and an understanding of how estimation can be used as a real life application. For this activity, it is expected that students have been introduced to rounding as a process for estimating.

Common Misconceptions: (4.NBT.4-6)

Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.

Specific strategies or students having difficulty with lining up similar place values in numbers as they are adding and subtracting.

Sometimes it is helpful to have them write their calculations on grid paper or lined notebook paper with the lines running vertical. This assists the student with lining up the numbers more accurately.

If students are having a difficult time with a standard addition algorithm, a possible modification to the algorithm might be helpful. Instead of the 'shorthand' of 'carrying,' students could add by place value, moving left to right placing the answers down below the 'equals' line. For example:

$$\begin{array}{r} 249 \\ 372 \\ 500 \\ 110 \\ \underline{+\ 11} \\ 621 \end{array}$$

(start with 200 + 300 to get the 500, then 40 + 70 to get 110, and 9 + 2 for 11)

Domain: Number and Operations in Base Ten (NBT)

***Cluster**: Use place value understanding and properties to perform multi-digit arithmetic.*

Standard: Grade 4.NBT.5

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4.NBT.4

Explanation and Examples:

Students who develop flexibility in breaking numbers apart (decomposing numbers) have a better understanding of the importance of place value and the distributive property in multi-digit multiplication.

Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication and understanding why it works, is an expectation in the 5th grade.

This standard calls for students to multiply numbers using a variety of strategies.

Example:

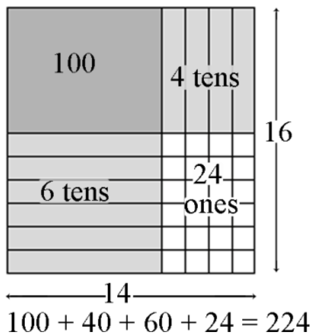
There are 25 dozen cookies in the bakery. What is the total number of cookies at the baker?

Student 1	Student 2	Student 3
25 x12 I broke 12 up into 10 and 2 and 25 x 10 = 250 25 x 2 = 50 250 +50 = 300	25 x 12 I broke 25 up into 5 groups of 5 5 x 12 = 60 I have 5 groups of 5 in 25 60 x 5 = 300	25 x 12 I doubled 25 and cut 12 in half to get 50 50 x 6 = 300

Use of place value and the distributive property are applied in the scaffold examples below.

- To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 X 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) = 600 + 300 + 24 = 924.
- The area model shows the partial products.

14 x 16 = 224



Using the area model, students first verbalize their understanding:

- 10 x 10 is 100
- 4 x 10 is 40
- 10 x 6 is 60, and
- 4 x 6 is 24.

They use different strategies to record this type of thinking.

- Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

- Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

  25
X 24
400 (20 x 20)
100 (20 x 5)
  80 (4 x 20)
  20 (4 x 5)
600

-

  25
X 24
 500 (20 x 25)
 100 (4 x 25)
 600

- **Matrix Model:** This model should be introduced after students have facility with the strategies shown above.

	20	5	
20	400	100	500
4	80	20	100
	480 +	120	600

Example:
What would an array area model of 74 X 38 look like?

	70	4
30	$70 \times 30 = 2{,}100$	$4 \times 30 = 120$
8	$70 \times 8 = 560$	$4 \times 8 = 32$

$$2{,}000 = 560 + 1{,}200 + 32 = 2{,}812$$

Instructional Strategies: **See Grade 4.NBT.4**

Tools/Resources
See: "Using Arrows to Multiply Bigger Numbers", Georgia Department of Education. In this task students demonstrate how to multiply two-digit numbers using arrays. Students will be given a multiplication problem with a two-digit number by a two-digit number. They will use graph paper to solve the problem by breaking it down into partial products (smaller arrays to find the answer).

For detailed information see Progressions for the Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten

Common Misconceptions: See Grade 4.NBT.4

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Use place value understanding a properties of operations to perform multi-digit operations.*

Standard: Grade 4.NBT 6

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4.NBT.4

Explanation and Examples:

In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Examples:

A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

- **Using Base 10 Blocks**: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
- **Using Place Value:** 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4)
- **Using Multiplication:** 4 x 50 = 200, 4 x 10 = 40, 4 x 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65

This standard calls for students to explore division through various strategies.

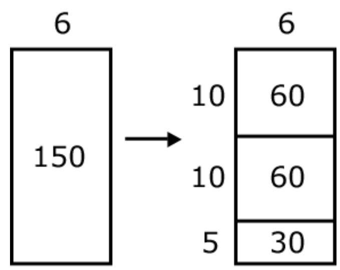
Student 1	**Student 2**	**Student 3**
592 divided by 8 There are 70 8's in 560 592 - 560 = 32 There are 4 8's in 32 70 + 4 = 74	592 divided by 8 I know that 10 8's is 80 If I take out 50 8's that is 400 592 - 400 = 192 I can take out 20 more 8's which is 160 192 - 160 = 32 8 goes into 32 4 times I have none left I took out 50, then 20 more, then 4 more That's 74 592 -400 | 50 192 - 160 | 20 32 -32 | 4 0	I want to get to 592 8 x 25 = 200 8 x 25 = 200 8 x 25 = 200 200 + 200 + 200 = 600 600 - 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams

Example:

Using an Open Array or Area Model

After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5^{th} grade.

$150 \div 6$



Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

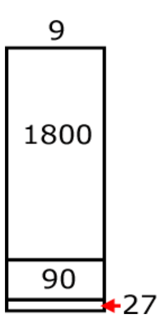
1. Students think, 6 times what number is a number close to 150? They recognize that 6 x 10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6 x 5 is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Students express their calculations in various ways:
   a.

$$\begin{array}{l} 150 \\ \underline{-60(6 \times 10)} \\ 90 \\ \underline{-60(6 \times 10)} \\ 30 \\ \underline{-30(6 \times 5)} \\ 0 \end{array} \qquad 150 \div 6 = 10 + 10 + 5 = 25$$

   b. $150 \div 6 = (60 \div 6) + (60 \div 6) + (30 \div 6) = 10 + 10 + 5 = 25$

Example:

1917×9



A student's description of his or her thinking may be:

I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines.

$1917 \div 9 = 213$

Instructional Strategies: **See Grade 4.NBT. 4**

Major Supporting Additional **Depth Opportunities(DO)**

Tools/Resources

For detailed information see Progressions for the Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten

Common Misconceptions: See Grade 4.NBT. 4

SCUSD 4th Grade Curriculum Map

Unit 1 - Computation with Whole Numbers, Place Value, & Rounding
Sequence of Learning Outcomes
1) Fluently add and subtract multi-digit whole numbers (up to 1,000) using various methods, such as decomposition and the distributive property of addition (NBT[1]) 4.NBT.4
Unit 2 - Whole Numbers: Multiplication and Division
Sequence of Learning Outcomes
1) Multiply two-digit by single-digit numbers progressing up to four-digit by single-digit numbers using contextual problems. Students use mental computation and rounding to assess the reasonableness of their solutions. 4.NBT.5
2) Use the area model to develop division strategies. Relate division back to multiplication with the area model. 4.NBT.6
3) Decompose larger dividends into smaller "like" base-ten units, related to distributive property (refer to CA Framework, pg. 20). 4.NBT.6

enVisionMATH Common Core Grade 4

Topic 4: Addition and Subtraction of Whole Numbers
Sequence of Learning Objectives **Lessons 4-3 – 4-6**
Lesson 4-3 – Adding Whole Numbers In this lesson, you will • Add numbers to hundreds and thousands with and without regrouping
Lesson 4-4 – Subtracting Whole Numbers In this lesson, you will • Subtract numbers to thousands with and without regrouping
Lesson 4-5 – Subtracting across Zeros In this lesson, you will • Subtract numbers with zeros to thousands
Lesson 4-6 – Problem Solving: Draw a Picture and Write an Equation In this lesson, you will • Use a picture or diagram to translate an addition or subtraction problem into a number sentence or equation
Topic 5: *Number Sense*: Multiplying by 1-Digit Numbers
Sequence of Learning Objectives **Lessons 5-1 – 5-5**
Lesson 5-1 – Arrays and Multiplying by 10 and 100 In this lesson, you will • Use arrays to multiply by 10 and 100
Lesson 5-2 – Multiplying by Multiple of 10 and 100 In this lesson, you will • Use basic multiplication facts and number patterns to multiply by multiples of 10 and 100
Lesson 5-3 – Breaking Apart to Multiply In this lesson, you will • Break apart numbers and use arrays to multiply
Lesson 5-4 – Using Mental Math to Multiply In this lesson, you will • Use compensation to multiply numbers mentally
Lesson 5-5 – Using Rounding to Estimate In this lesson, you will • Use rounding to estimate solutions to multiplication problems
Lesson 5-6 – Problem Solving: Reasonableness In this lesson, you will • Check for reasonableness by making sure their calculation answer the questions asked and by using estimation to make sure the calculation was performed correctly
Topic 6: *Developing Fluency:* Multiplying by 1-Digit Numbers
Sequence of Learning Objectives **Lessons 6-1 – 6-6**

Lesson 6-1 – Arrays and Using an Expanded Algorithm In this lesson, you will • Record multiplication using an expanded algorithm
Lesson 6-2 – Connecting the Expanded and Standard Algorithms In this lesson, you will • Multiply 2-digit numbers by 1-digit numbers using paper-and-pencil methods
Lesson 6-3 – Multiplying 2-Digit by 1-Digit Numbers In this lesson, you will • Multiply 2-digit by 1-digit numbers using the standard algorithm and estimate to check for reasonableness
Lesson 6-4 – Multiplying 3- and 4-Digit by 1-Digit Numbers In this lesson, you will • Use the standard algorithm to multiply 3- and 4-digit numbers by 1-digit numbers.
Lesson 6-5 – Multiplying by 1-Digit numbers In this lesson, you will • Multiply 2-, 3-, and 4-digit numbers by 1-digit numbers using the standard algorithm and estimate to check for reasonableness
Topic 7: *Number Sense*: Multiplying by 2-Digits Numbers
Sequence of Learning Objectives Lessons 7-1 – 7-4
Lesson 7-1 – Arrays and Multiplying 2-Digit Numbers by Multiples of 10 In this lesson, you will • Use arrays to multiply 2-digit numbers by multiples of 10
Lesson 7-2 – Using Mental Math to Multiply 2-Digit Numbers In this lesson, you will • Discover and use patterns to multiply by multiples of 10
Lesson 7-3 – Using Rounding to Estimate In this lesson, you will • Use rounding to estimate solutions to multiplication problems involving two 2-digit numbers.
Lesson 7-4 – Using Compatible Numbers to Estimate In this lesson you will • Use compatible numbers and rounding to estimate solutions to multiplication problems involving two 2-digit numbers
Topic 8: *Developing Fluency*: Multiplying by 2-Digits Numbers
Sequence of Learning Objectives Lessons 8-1 – 8-4
Lesson 8-1 – Arrays and Multiplying 2-Digit Numbers In this lesson, you will • Use arrays to multiply two-digit numbers by two-digit numbers to find the product
Lesson 8-2 – Arrays and an Expanded Algorithm In this lesson, you will • Use an expanded algorithm to multiply two-digit numbers by two-digit numbers to find the product

Lesson 8-3 – Multiplying 2-Digit Numbers by Multiples of 10
In this lesson, you will

- Use grids and patterns to multiply two-digit numbers and multiples of 10

Lesson 8-4 – Multiplying 2-Digit by 2-Digit Numbers
In this lesson, you will

- Use partial products to multiply two-digit numbers by two-digit numbers and find the product

Topic 9: *Number Sense*: Dividing by 1-Digit Divisors

Sequence of Learning Objectives
Lessons 9-4 – 9-6

Lesson 9-1 – Using Mental Math to Divide
In this lesson, you will

- Use basic facts and patterns of zeros to solve division problems with 3-digit dividends and 1-digit divisors

Lesson 9-2 – Estimating Quotients
In this lesson, you will

- Use compatible numbers and rounding to estimate quotients

Lesson 9-3 – Estimating Quotients for Greater Dividends
In this lesson, you will

- Estimate quotients of multi-digit division problems using multiplication facts and place-value concepts

Lesson 9-4 – Dividing with Remainders
In this lesson, you will

- Divide whole numbers by 1-digit divisors resulting in quotients with remainders

Lesson 9-5 – Multiplication and Division Stories
In this lesson, you will

- Use words and models to represent multiplication and division problems accurately

Lesson 9-6 – Problem Solving: Draw a Picture and Write an Equation
In this lesson, you will

- Draw pictures and write related number sentences to solve problems

Topic 10: *Developing Fluency*: Dividing by 1-Digit Divisors

Sequence of Learning Objectives
Lessons 10-1 – 10-6

Lesson 10-1 – Division as Repeated Subtraction
In this lesson, you will

- Record division as repeated subtraction

Lesson 10-2 – Using Objects to divide: Division as Sharing
In this lesson, you will

- Use place value to understand the algorithm of long division

Lesson 10-3 – Dividing 2-Digit by 1-Digit Numbers
In this lesson, you will

- Use the standard algorithm to divide a two-digit number by a one-digit number

Lesson 10-4 – Dividing 3-Digit by 1-Digit Numbers
In this lesson, you will

- Use the standard algorithm to divide 3-digit numbers by 1-digit numbers

Lesson 10-5 – Deciding Where to Start Dividing In this lesson, you will • Use the standard algorithm to divide 3-digit numbers by 1-digit numbers and properly decide where to begin dividing
Lesson 10-6 – Dividing 4-Digit by 1-Digit Numbers In this lesson, you will • Estimate and find quotients for 4-digit dividends and 1-digit divisors

Name ____________________

Mark the best answer.

1. There are 4,800 children who go to school in Grades 1–8 in the town of Warren. How many children are in each grade if the number in each is equal? (9-1)

A 60
B 600
C 6,000
D 60,000

2. What is the quotient? (9-4)

$$\begin{array}{r} \square\,\text{R}\,\square \\ 4\overline{)39} \\ -\;\square\square \\ \hline \square \end{array}$$

A 3 R9
B 8 R3
C 8 R7
D 9 R3

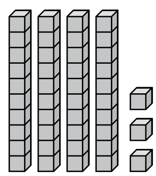
3. The play is performed 7 times. A total of 1,585 tickets were sold and the same number of people attended each performance. About how many people attended each performance? (9-3)

A 300
B 200
C 150
D 100

4. Is the quotient of these division sentences 8 R5? Mark *Yes* or *No*. (9-4)

53 ÷ 6	**A** Yes	**B** No
56 ÷ 7	**A** Yes	**B** No
59 ÷ 6	**A** Yes	**B** No
61 ÷ 7	**A** Yes	**B** No

5. Deanna has 43 ceramic tiles to make a decorative pattern on her kitchen floor. She will use the same number of tiles in each corner of the floor. She will use any remaining tiles to make a design in the middle. How many tiles can she use in each corner, and how many tiles will she have left for the middle? (9-5)



A Each corner will have 11 tiles. There will be 3 left over.
B Each corner will have 10 tiles. There will be 3 left over.
C Each corner will have 9 tiles There will be 14 left over.
D Each corner will have 9 tiles. There will be 0 left over.

Name ______________________________

6. Miguel spent $207 on 7 model airplane kits. Which number sentence shows the best way to estimate the amount he spent for each kit? (9-2)

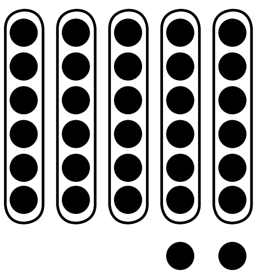
A $\$140 \div 7 = \$20$

B $\$210 \div 7 = \$30$

C $7 \times \$200 = \$1{,}400$

D $7 \times \$210 = \$1{,}470$

7. Mrs. Lincoln steamed 32 clams for a family picnic. There were 5 people eating clams and each person ate an equal number of clams. How many clams were left over? (9-5)



A 6 left

B 5 left

C 2 left

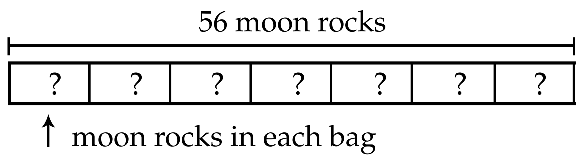
D 1 left

8. A jeweler made 96 necklaces. She put an equal number of necklaces in each of 5 display trays. How many necklaces are in each tray? How many remaining necklaces are not displayed? (9-4)

____

____

9. An astronaut collected 56 moon rocks. She has 7 bags to put them in. Write a number sentence that shows how many moon rocks she can put in each bag if she puts the same number in each bag. (9-6)



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10. A case of toothpicks has 5,400 toothpicks. There are 9 boxes of toothpicks in the case. How many toothpicks are in each box? (9-1)

____

11. Estimate the quotient for $627 \div 9$. Explain how you found your answer. (9-2)

____

____

____

Name ______________________________

12. **Writing to Explain** Tyler has 83 football cards that he wants to put into an album. Each page holds 6 cards. How many pages will he need? How many spaces will he have left for new cards? Explain your answer. (9-5)

13. Nick uses 8 dowels to make one birdhouse. If he bought 1,581 dowels, about how many birdhouses will he be able to make? Explain. (9-3)

14. What number sentence comes next in the pattern? (9-1)

$21 \div 7 = 3$

$210 \div 7 = 30$

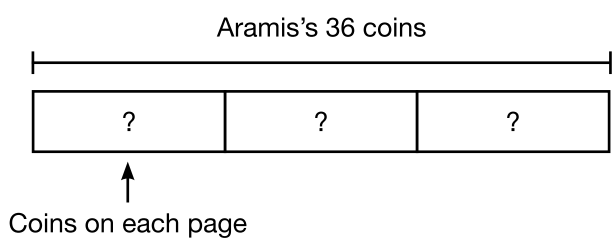
$2{,}100 \div 7 = 300$

15. There are 18 people waiting for a ride. A car holds 4 people. How many cars are needed? (9-4)

16. A box has 640 nails. Each model boat needs 8 nails to hold it together. How many model boats can be made? (9-1)

17. Casey is saving to buy a new computer that costs $2,450. She saves an equal amount of money each month for 5 months. About how much does she need to save each month to buy the computer? (9-3)

18. Aramis has 36 coins that he wants to display on 3 pages in his coin album. Write a number sentence that shows how many coins he can put on each page. (9-6)



Assessment Options from Illustrative Mthematics

Illustrative Mathematics

4.NBT Mental Division Strategy

Jillian says

I know that 20 times 7 is 140 and if I take away 2 sevens that leaves 126. So 126 ÷ 7 = 18.

a. Is Jillian's calculation correct? Explain.

b. Draw a picture showing Jillian's reasoning.

c. Use Jillian's method to find 222 ÷ 6.

Illustrative Mathematics

4.NBT Millions and Billions of People

Historians estimate that there were about 7 million people on the earth in 4,000 BCE. Now there are about 7 billion!

We write 7 million as 7,000,000.

We write 7 billion as 7,000,000,000.

How many times more people are there on the earth now than there were in 4,000 BCE?