



Curriculum  
Map

# Common Core Mathematics: High School Math 1

Sacramento City Unified School District

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<b>High School Math 1 Year-at-a-Glance</b>			
	<b>Month</b>	<b>Unit</b>	<b>Content Standards</b>
<b>District Benchmark 1</b>	September/October	<b>Unit #1</b> Relationships Between Quantities	A.SSE.1 N.Q.1, 2, 3 A.CED.1, 2, 3, 4 A.REI.1, 3, 3.1
	October/November	<b>Unit #2</b> Systems of Equations and Inequalities	A.REI.5, 6, 12 A.CED.3
<b>District Benchmark 2</b>	November	<b>Unit #3</b> Connecting Algebra and Geometry Through Coordinates	G.GPE.4, 5, 7
	December/January	<b>Unit #4</b> Understanding and Analyzing Functions	A.REI.10, 11 F.LE.1a, 3, 5 F.IF.1, 2, 3, 4, 5, 6, 7a, 7e, 9
<b>District Benchmark 3</b>	February/March	<b>Unit #5</b> Building Functions	F.BF.1, 2, 3 F.LE.1b, 1c, 2
	March/April	<b>Unit #6</b> Descriptive Statistics	S.ID.1, 2, 3, 5, 6, 7, 8, 9
<b>CAASPP (Smarter Balanced Summative Test)</b>	May/June	<b>Unit #7</b> Congruence and Constructions	G.CO.1, 2, 3, 4, 5, 6, 7, 8, 12, 13

## Unit #1: Relationships Between Quantities

(Approx. # Days - 22)

Content Standards:

A.SSE.1; N.Q.1, 2, 3; A.CED.1, 2, 3, 4; A.REI.1, 3, 3.1

### Math Common Core Content Standards:

#### Conceptual Category: Algebra

#### Domain: Seeing Structure in Expressions A-SSE

**Interpret the structure of expressions.** [In Mathematics I, these standards address linear expressions and exponential expressions with integer exponents.]

1. Interpret expressions that represent a quantity in terms of its context. ★
  - a. Interpret parts of an expression, such as terms, factors, and coefficients. ★
  - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1 + r)^n$  as the product of  $P$  and a factor not depending on  $P$ .* ★

#### Conceptual Category: Number and Quantity

#### Domain: Quantities N-Q

**Reason quantitatively and use units to solve problems.**

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
2. Define appropriate quantities for the purpose of descriptive modeling. ★
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★

#### Conceptual Category: Algebra

#### Domain: Creating Equations A-CED

**Create equations that describe numbers or relationships.** [In Mathematics I, these standards address linear equations and exponential equations with integer inputs only. For A.CED.3, linear equations only.]

1. Create equations and inequalities in one variable **including ones with absolute value** and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ★ [In Mathematics I, this standard addresses linear and exponential integer inputs]
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ [In Mathematics I, this standard addresses linear and exponential integer inputs]
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★ [In Mathematics I, this standard addresses linear integer inputs]
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .* ★

#### Domain: Reasoning with Equations and Inequalities A-REI

**Understand solving equations as a process of reasoning and explain the reasoning.**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**Solve equations and inequalities in one variable.**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- 3.1 Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context. (CA)

**Standards for Mathematical Practice:**

1. Make Sense of Problems and Persevere in Solving Them
2. Reason Abstractly and Quantitatively
3. Construct Viable Arguments and Critique the Reasoning of Others
4. Model with Mathematics
5. Use Appropriate Tools Strategically
6. Attend to Precision
7. Look For and Make Use of Structure
8. Look For and Express Regularity in Repeated Reasoning

**ELD Standards to Support Unit**

[Add text]

**SEL Competencies:**

[Add text]

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Outcomes	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
<ul style="list-style-type: none"> <li>• What are the similarities and differences between a linear equation and an exponential equation?</li> <li>• What is absolute value? What are some real-life situations that require equations involving absolute value?</li> <li>• What are some similarities and differences between the graphs of linear equations, linear inequalities, equations involving absolute value, and exponential equations?</li> <li>• Why is it useful to interpret parts of an equation in relation to real-world context?</li> <li>• When would you use a number line to graph a solution and when would you use a coordinate plane to graph a solution?</li> <li>• What kinds of situations require you to use inequalities?</li> <li>• What kinds of situations require you to use exponential equations?</li> </ul>	<p>Assessments/Tasks aligned to learning experiences:</p> <p>For learning experiences 1-3:  <a href="http://map.mathshell.org/materials/tasks.php?taskid=286#task286">http://map.mathshell.org/materials/tasks.php?taskid=286#task286</a>  <a href="http://map.mathshell.org/materials/lessons.php?taskid=221#task221">http://map.mathshell.org/materials/lessons.php?taskid=221#task221</a>  <a href="http://map.mathshell.org/materials/lessons.php?taskid=554#task554">http://map.mathshell.org/materials/lessons.php?taskid=554#task554</a>  <a href="https://www.illustrativemathematics.org/illustrations/1850">https://www.illustrativemathematics.org/illustrations/1850</a>  <a href="https://www.illustrativemathematics.org/illustrations/613">https://www.illustrativemathematics.org/illustrations/613</a></p> <p>For learning experience 4:  <a href="https://www.illustrativemathematics.org/illustrations/1828">https://www.illustrativemathematics.org/illustrations/1828</a></p> <p>For learning experiences 5 – 6:  <a href="https://www.illustrativemathematics.org/illustrations/1351">https://www.illustrativemathematics.org/illustrations/1351</a>  <a href="https://www.illustrativemathematics.org/illustrations/807">https://www.illustrativemathematics.org/illustrations/807</a></p> <p>For learning experiences 7 – 9:</p> <p>For learning experiences 10 – 11:</p>	<p>Students will be able to...</p> <ol style="list-style-type: none"> <li>1) Make sense of various parts of a given linear expression (such as its terms, factors, and coefficients) that models a real-life situation. (Framework p. 18)</li> <li>2) Create linear equations given a real-world context, and interpret parts of the equation (such as its terms, factors, and coefficients) in terms of the situation it models.</li> <li>3) Explain each step in solving a simple linear equation in one variable and construct a viable argument to justify solutions.</li> <li>4) Rearrange formulas to highlight a specific quantity, using the same reasoning as in solving equations.</li> <li>5) Create inequalities in one variable, and interpret parts of the inequality in terms of the situation it models, with a focus on what the inequality symbol means in the context of the situation.</li> <li>6) Solve one-variable inequalities (including absolute value inequalities) from real-world and mathematical problems. Graph the solution on a number line and interpret the solution in terms of the context.</li> <li>7) Create absolute value equations to represent real-world situations, and interpret parts of the equation in terms of the situation it models.</li> <li>8) Understand the definition of absolute value from:             <math display="block"> x  = \begin{cases} -x, &amp; x &lt; 0 \\ x, &amp; x \geq 0 \end{cases}</math> </li> <li>9) Use the definition of absolute value to solve one-variable equations involving absolute value from real-world and mathematical problems. Graph the solution on a number line and interpret the solution in terms of the context.</li> <li>10) Make sense of various parts of a given exponential</li> </ol>	<p>Rational numbers: Because students have experience with rational numbers in previous grades, include within all experiences  <a href="#">Solving linear equations: Google Doc</a></p> <p>This unit mostly focuses on linear equations; students will learn more about creating and graphing exponential equations and exponential functions as they continue through Unit 3 and Unit 4.</p> <p>Experiences 1-11 are focused on equations and inequalities in one variable. Experience 12 focuses on equations in two variables and serves as the transition to Unit 2.</p> <p>Working definition of absolute value: Students have previously developed an understanding of absolute value as the distance from zero</p>		<p><a href="#">CA Mathematics Framework Math 1 p. 15 – 22</a></p> <p><a href="#">Progressions for the Common Core – High School, Algebra</a></p> <p><a href="#">Progressions for the Common Core – Modeling, HS</a></p> <p><a href="#">North Carolina Unpacked Content, HS Alg: p. 2, 10 – 13</a></p> <p><a href="#">High School CCSS Flip Book</a></p>

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Outcomes	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
		<p>expression with an integer exponent (such as its base, exponent, and coefficient) that models a real-life situation. (Framework p. 18)</p> <p>11) Solve simple exponential equations in one variable by inspection, for example <math>2^x = 8</math> and verify solutions through substitution.</p> <p>12) Graph linear equations on a coordinate plane that represent real-world and mathematical problems. Identify parts of the graph that make sense in terms of the situation it models (i.e. represent constraints).</p>	<p>Graphing linear equations in slope-intercept form and standard form: Students have experience in graphing on both forms, with multiple strategies, in previous grades.</p>		

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## Unit #2: Systems of Equations and Inequalities

(Approx. # Days - 18)

Content Standards:

A.REI.5, 6,12

A.CED.3

### Math Common Core Content Standards:

#### Conceptual Category: Algebra

#### Domain: Reasoning with Equations and Inequalities A-REI

##### Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

##### Represent and solve equations and inequalities graphically.

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

#### Domain: Creating Equations A-CED

##### Create equations that describe numbers or relationships.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★ [In Mathematics I, this standard addresses linear integer inputs]

#### Standards for Mathematical Practice:

1. Make Sense of Problems and Persevere in Solving Them
2. Reason Abstractly and Quantitatively
4. Model with Mathematics
5. Use Appropriate Tools Strategically
6. Attend to Precision
8. Look For and Express Regularity in Repeated Reasoning

#### ELD Standards to Support Unit

[Add text]

#### SEL Competencies:

[Add text]



Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
<ul style="list-style-type: none"> <li>• What does the point of intersection mean of a graph of a system of two linear equations?</li> <li>• Why does it make sense that two equations that form parallel lines have no solution?</li> <li>• Why does it make sense that two equations that form the same line have infinite solutions?</li> <li>• How can you use a table to represent a system of linear equations and to find/estimate its solution?</li> <li>• Without graphing or solving algebraically, how can you determine the number of solutions to a system of linear equations?</li> <li>• How do you determine the most efficient method for graphing a linear equation?</li> <li>• How do you determine the most efficient method for solving (algebraically) a system of two linear equations?</li> <li>• When graphing equations to represent a real-world context, how do you label your axes with the appropriate variables?</li> <li>• When modeling a real-world situation, when might you write an equation in slope-intercept</li> </ul>	<p>Assessments/Tasks aligned to learning experiences:</p> <p>For learning experiences 1 – 7 (Systems of equations in a <b>real-world</b> context):</p> <p><a href="http://www.illustrativemathematics.org/illustrations/462">http://www.illustrativemathematics.org/illustrations/462</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/761">http://www.illustrativemathematics.org/illustrations/761</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/935">http://www.illustrativemathematics.org/illustrations/935</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/936">http://www.illustrativemathematics.org/illustrations/936</a></p> <p>For learning experiences 1 – 7 (Systems of equations in a <b>mathematical</b> context):</p> <p><a href="http://www.illustrativemathematics.org/illustrations/1033">http://www.illustrativemathematics.org/illustrations/1033</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/1363">http://www.illustrativemathematics.org/illustrations/1363</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/1833">http://www.illustrativemathematics.org/illustrations/1833</a></p> <p>For learning experiences 8 – 9 (Systems of inequalities):</p> <p>Real-world context:</p> <p><a href="http://www.illustrativemathematics.org/illustrations/644">http://www.illustrativemathematics.org/illustrations/644</a></p> <p>Mathematical context:</p> <p><a href="http://www.illustrativemathematics.org/illustrations/1205">http://www.illustrativemathematics.org/illustrations/1205</a></p>	<p>Students will be able to...</p> <ol style="list-style-type: none"> <li>1) Solve a system of equations in slope-intercept form resulting from a real-world or mathematical context, both exactly and approximately, by graphing and by using a table to show the relationship between the two models. Analyze parts of the graph in context.</li> <li>2) Solve systems of linear equations in slope-intercept form by setting the two equations equal to each other (substitution) for the purpose of showing that the solution is where the lines intersect and where the equations are equal.</li> <li>3) Graph a system of two linear equations given a real-world context where there is either no solution or infinite solutions. Analyze what “no solution” and “infinite solutions” mean both in terms of the graph, the equations, and the situations they model.</li> <li>4) Solve a system of equations in standard form by graphing, in terms of a real-world or mathematical context, both exactly and approximately. Analyze parts of the graph in context.</li> <li>5) Explain why the sum or difference of two linear equations results in an equation that produces a line that passes through the point of intersection of the original system of equations.*</li> <li>6) Use the method of elimination to solve systems of linear equations in standard form resulting from real-world and mathematical contexts.</li> <li>7) Solve systems of linear equations in any form, using methods of substitution or elimination, resulting from real-world or mathematical contexts and make meaning of the solution in terms of the context.</li> <li>8) Graph a linear inequality in two variables on the coordinate plane, given a real-world or mathematical</li> </ol>	<p>Video Links:</p> <p><i>The effects of multiplying an equation by a constant</i></p> <p><a href="http://learnzillion.com/lessons/3754-understand-the-effects-of-multiplying-an-equation-by-a-constant">http://learnzillion.com/lessons/3754-understand-the-effects-of-multiplying-an-equation-by-a-constant</a></p> <p>*This proves the validity of the elimination/addition method. See video, <i>Adding equations in a system of equations</i></p> <p><a href="http://learnzillion.com/lessons/720-add-equations-in-a-system-of-equations">http://learnzillion.com/lessons/720-add-equations-in-a-system-of-equations</a></p>		<p><i>CA Mathematics Framework Math 1</i> p. 22 – 25</p> <p><a href="http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf">http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf</a></p> <p><i>Progressions for the Common Core – High School, Algebra</i></p> <p><a href="http://commoncoretools.me/wp-content/uploads/2013/07/css_progression_algebra_2013_07_03.pdf">http://commoncoretools.me/wp-content/uploads/2013/07/css_progression_algebra_2013_07_03.pdf</a></p> <p>North Carolina Unpacked Content, HS Algebra: pgs. 15, 18</p> <p><a href="http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/algebra.pdf">http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/algebra.pdf</a></p> <p><i>High School CCSS Flip Book</i></p> <p><a href="http://katm.org/wp/">http://katm.org/wp/</a></p>

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
<p>form and when might you write an equation in standard form?</p> <ul style="list-style-type: none"> <li>• Why does the sum or difference of two linear equations result in an equation that produces a line that passes through the point of intersection of the original system of equations?</li> <li>• How do you represent a solution to a linear inequality in two variables?</li> <li>• What are the meanings of the shaded region and the boundary line in the solution to a linear inequality or system of linear inequalities?</li> <li>• What is the difference between a solid boundary line and a dashed boundary line in the graph of a linear inequality or system of linear inequalities?</li> </ul>		<p>context. Understand that the points within the shaded region (half-plane) are the solution to the inequality, and make sense of the solution in the context of the problem.</p> <p>9) Solve a system of linear inequalities by graphing, given a real-world or mathematical context, and interpret the points within the shaded region in terms of the context of the problem.</p>			<p><a href="https://www.scsd.org/wp-content/uploads/2012/05/High-School-CCSS-Flip-Book-USD-259-2012.pdf">wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf</a></p>

## Unit #3: Connecting Algebra and Geometry Through Coordinates

(Approx. # Days- 8)

Content Standards: G.GPE.4, 5, 7

### Math Common Core Content Standards:

#### Conceptual Category: Geometry

#### Domain: Expressing Geometric Properties with Equations G-GPE

**Use coordinates to prove simple geometric theorems algebraically.** [In Mathematics I, these standards include the distance formula and its relation to Pythagorean Theorem.]

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0, 2)$ .*
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★

### Standards for Mathematical Practice:

1. Make Sense of Problems and Persevere in Solving Them
3. Construct Viable Arguments and Critique the Reasoning of Others
7. Look For and Make Use of Structure
8. Look For and Express Regularity in Repeated Reasoning

### ELD Standards to Support Unit

[Add text]

### SEL Competencies:

[Add text]

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
<ul style="list-style-type: none"> <li>Why are the slopes of parallel lines equal?</li> <li>Why do the slopes of perpendicular lines have a product of <math>-1</math>?</li> <li>How is the distance formula related to the Pythagorean Theorem?</li> <li>When can you find the distance between two points <i>without</i> necessarily using the distance formula? When do you <i>need</i> to use the distance formula or Pythagorean Theorem to find the distance between two points?</li> </ul>	<p>Assessments/Tasks aligned to learning experiences:</p> <p>1) <a href="http://map.mathshell.org/materials/lessons.php?taskid=226#task226">http://map.mathshell.org/materials/lessons.php?taskid=226#task226</a></p> <p>2) <a href="http://map.mathshell.org/materials/tasks.php?taskid=270#task270">http://map.mathshell.org/materials/tasks.php?taskid=270#task270</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/1347">http://www.illustrativemathematics.org/illustrations/1347</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/1302">http://www.illustrativemathematics.org/illustrations/1302</a></p> <p>4) <a href="http://www.illustrativemathematics.org/illustrations/1684">http://www.illustrativemathematics.org/illustrations/1684</a></p>	<p>Students will be able to...</p> <ol style="list-style-type: none"> <li>Discover that the slopes of parallel lines are equal and that the slopes of perpendicular lines have a product of <math>-1</math>, through an investigative approach. (Framework p. 30 – 31)</li> <li>Use their understanding of parallel and perpendicular lines and given coordinate points to prove simple geometric theorems algebraically, for example that a figure defined by four points is a rectangle because the lines containing opposite sides of the figure are parallel and the lines containing adjacent sides are perpendicular.</li> <li>Use the Pythagorean Theorem to derive the distance formula: <math>d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}</math></li> <li>Use given coordinate points and the distance formula to compute perimeters of polygons and areas of triangles and rectangles, in mathematical problems and real-life situations.</li> </ol>	<p>An intuitive argument for why parallel lines have the same slope might read: “Since the two lines never meet, each line must <i>keep up</i> with the other as we travel along the slopes of the lines. So it seems obvious that their slopes must be equal” (Framework p.30).</p> <p>Perpendicular relationships of lines can be represented by rotating a right triangle 90 degrees around one of its vertices (Framework p. 30-31).</p> <p><a href="#">Deriving the distance formula from the Pythagorean Theorem</a></p>		<p><i>CA Mathematics Framework Math 1</i> p. 29 – 31  <a href="http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf">http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf</a></p> <p>North Carolina Unpacked Content, HS Geometry: pg. 18 – 19  <a href="http://www.ncpublicschools.org/docs/academic/standards/communication-core-tools/unpacking/math/geometry.pdf">http://www.ncpublicschools.org/docs/academic/standards/communication-core-tools/unpacking/math/geometry.pdf</a></p> <p><i>High School CCSS Flip Book</i>  <a href="http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf">http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf</a></p>

## Unit #4: Understanding and Analyzing Functions

(Approx. # Days - 30)

Content Standards:

A.REI.10, 11

F.LE.1a, 3, 5

F.IF.1, 2, 3, 4, 5, 6, 7a, 7e, 9

### Math Common Core Content Standards:

#### Conceptual Category: Algebra

##### Domain: Reasoning with Equations and Inequalities A-REI

**Represent and solve equations and inequalities graphically.** [Linear and exponential; learn as general principle.]

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★ [In Mathematics I, this standard addresses linear and exponential equations and call for an understanding of their graphs as a general principle.]

#### Conceptual Category: Functions

##### Domain: Linear, Quadratic, and Exponential Models F-LE

**Construct and compare linear, quadratic, and exponential models and solve problems.** [Linear and exponential.]

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. ★
  - a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ★
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★ [Linear and exponential functions only.]

##### Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

##### Domain: Interpreting Functions F-IF

**Understand the concept of a function and use function notation.** [Learn as general principle. Focus on linear and exponential (integer domains) and on arithmetic and geometric sequences.]

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1$ ,  $f(n + 1) = f(n) + f(n - 1)$  for  $n \geq 1$ .*

**Interpret functions that arise in applications in terms of the context** [Linear and exponential functions. For F.1F.6, focus on linear functions and intervals for exponential functions whose domain is a subset of integers.]

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★*

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function. ★*
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

**Analyze functions using different representations.** [In Mathematics I, these standards address linear and exponential functions.]

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
  - a. Graph linear and quadratic functions and show intercepts, maxima, and minima. ★
  - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ★
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

**Standards for Mathematical Practice:**

1. Make Sense of Problems and Persevere in Solving Them
2. Reason Abstractly and Quantitatively
3. Construct Viable Arguments and Critique the Reasoning of Others
4. Model with Mathematics
5. Use Appropriate Tools Strategically
6. Attend to Precision
7. Look For and Make Use of Structure
8. Look For and Express Regularity in Repeated Reasoning

**ELD Standards to Support Unit**

[Add text]

**SEL Competencies:**

[Add text]

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
<ul style="list-style-type: none"> <li>In a real-world situation, how do you determine which variable is dependent and which one is independent?</li> <li>What kinds of relationships are functions? What kinds of relationships are <i>not</i> functions?</li> <li>What is function notation and how is it similar to and different than an equation in two variables?</li> <li>Where can you find the domain and range of a function given a table, graph, or equation?</li> <li>How do you define an appropriate domain and range given a real-world context?</li> <li>Is it easier for you to identify the domain and range from a table or from a graph?</li> <li>What are the differences between linear and exponential functions?</li> <li>How can you identify whether a function is linear or exponential, given its graph, table, or related sequence?</li> <li>What are key features of the graph of an exponential function, and what are the key features of the graph of a linear function?</li> <li>How are arithmetic sequences related to linear functions, and how are geometric sequences</li> </ul>	<p>Assessments/Tasks aligned to learning experiences:</p> <p>Learning experiences 2 – 5:  <a href="http://www.illustrativemathematics.org/illustrations/243">http://www.illustrativemathematics.org/illustrations/243</a>  <a href="http://www.illustrativemathematics.org/illustrations/624">http://www.illustrativemathematics.org/illustrations/624</a>  <a href="http://www.illustrativemathematics.org/illustrations/635">http://www.illustrativemathematics.org/illustrations/635</a>  <a href="http://www.illustrativemathematics.org/illustrations/589">http://www.illustrativemathematics.org/illustrations/589</a>  <a href="http://www.illustrativemathematics.org/illustrations/588">http://www.illustrativemathematics.org/illustrations/588</a>  <a href="http://www.illustrativemathematics.org/illustrations/630">http://www.illustrativemathematics.org/illustrations/630</a>  <a href="http://www.illustrativemathematics.org/illustrations/634">http://www.illustrativemathematics.org/illustrations/634</a></p> <p>Learning experiences 7 – 8:  <a href="http://www.illustrativemathematics.org/illustrations/387">http://www.illustrativemathematics.org/illustrations/387</a>  <a href="http://www.illustrativemathematics.org/illustrations/637">http://www.illustrativemathematics.org/illustrations/637</a>  <a href="http://www.illustrativemathematics.org/illustrations/650">http://www.illustrativemathematics.org/illustrations/650</a>  <a href="http://www.illustrativemathematics.org/illustrations/650">http://www.illustrativemathematics.org/illustrations/650</a></p>	<p>Students will be able to...</p> <ol style="list-style-type: none"> <li>Graph linear and exponential equations in 2 variables on a coordinate plane, focusing on the line or curve as the set of all solutions to the equation.</li> <li>Given a linear or exponential equation that models a real-world situation, explore some of its properties through a table and/or graph. For example, draw attention to the various inputs and outputs of the equation, and interpret the equation in terms of a function (i.e. “___ is a function of ___”).</li> <li>Understand the definition of a function as for every input value, <math>x</math>, there is exactly one output value, <math>f(x)</math>. Use function notation and evaluate functions for given inputs, for example, given <math>f(x) = 3x + 4</math> find <math>f(2)</math>.</li> <li>Understand and use the vocabulary of “domain” (the set of input values) and “range” (the set out output values) with mathematical and real-world problems.*</li> <li>Given a linear or exponential function and its domain (in mathematical and real-world problems), create a table and graph that represents the function. Interpret the domain and range of the function from the table and/or graph in terms of the situation it models.</li> <li>Compare and contrast two functions that look similar but have different domains, in mathematical and real-world problems. For example, compare <math>f(x) = 2x + 3</math> where <math>f</math> has a domain of all real numbers to <math>g(n) = 2n + 3</math> where <math>g</math> has a domain of all integers.</li> <li>Given a linear or exponential function that models a real-world situation, infer what the domain of the function is from the situation it models. Graph the function and interpret the range of the function.</li> <li>Given a linear or exponential function that models a real-world situation, graph the function and interpret</li> </ol>	<p>This is the first time students will be introduced to function notation, <math>f(x)</math>, and functional vocabulary like domain and range.</p> <p>*Use real-world contexts to introduce domain and range, and then move to more abstract situations, including domain and range of arithmetic and geometric sequences. Students will know and understand the definitions of domain and range and will be able to apply them to any function.</p> <p>Example of an arithmetic sequence:            1, 4, 7, 10, ... ; which can be modeled by the linear function <math>f(x) = 3x - 2</math> whose domain is integers greater than or equal to 1.</p>		<p><i>CA Mathematics Framework Math 1</i>            p. 7–10, 13–15, 23–25  <a href="http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf">http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf</a></p> <p><i>Progressions for the Common Core – Gr. 8 and High School, Functions</i>  <a href="http://commoncoretools.org/wp-content/uploads/2013/07/css_progression_functions_2013_07_02.pdf">http://commoncoretools.org/wp-content/uploads/2013/07/css_progression_functions_2013_07_02.pdf</a></p> <p><i>Progressions for the Common Core – Modeling, High School</i>  <a href="http://commoncoretools.org/wp-content/uploads/2013/07/css_progression_modeling_2013_07_04.pdf">http://commoncoretools.org/wp-content/uploads/2013/07/css_progression_modeling_2013_07_04.pdf</a></p> <p>North Carolina Unpacked Content, HS Functions: pg. 2-7,</p>

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources										
<p>related to exponential functions?</p> <ul style="list-style-type: none"> <li>• How is the average rate of change over an interval of an exponential function similar/different to the average rate of change of a linear function?</li> <li>• How can you use functions to solve equations in one variable, for example <math>2x + 5 = 3x - 4</math>?</li> <li>• How can you use functions to approximate solutions for equations in one variable that you cannot solve by hand, for example <math>2^x = 5x + 7</math>?</li> </ul>	<p><a href="http://www.illustrativemathematics.org/illustrations/631">cs.org/illustrations/631</a>  <a href="http://www.illustrativemathematics.org/illustrations/639">http://www.illustrativemathematics.org/illustrations/639</a></p> <p>Learning experiences 10 – 11:  <a href="http://www.illustrativemathematics.org/illustrations/1500">http://www.illustrativemathematics.org/illustrations/1500</a>  <a href="http://www.illustrativemathematics.org/illustrations/686">http://www.illustrativemathematics.org/illustrations/686</a></p> <p>Learning experience 12:  <a href="http://www.illustrativemathematics.org/illustrations/362">http://www.illustrativemathematics.org/illustrations/362</a>  <a href="http://www.illustrativemathematics.org/illustrations/363">http://www.illustrativemathematics.org/illustrations/363</a>  <a href="http://www.illustrativemathematics.org/illustrations/629">http://www.illustrativemathematics.org/illustrations/629</a>  <a href="http://www.illustrativemathematics.org/illustrations/368">http://www.illustrativemathematics.org/illustrations/368</a></p>	<p>key features of the graph as they relate to the context, including intercepts, intervals on which the function is increasing/decreasing, maxima, minima, and end behavior.</p> <p>9) Understand that sequences are functions whose domain is a subset of integers greater than or equal to 1. Distinguish between arithmetic sequences (which can be modeled by linear functions) and geometric sequences (which can be modeled by exponential functions).</p> <p>10) Given a function or a table that represents a mathematical or real-world situation, find and interpret the average rate of change between two given points of an exponential function.</p> <p>11) Compare average rates of change over the same intervals from a linear function and from an exponential function. For example, how do the average rates of change of <math>f(x) = 3x + 4</math> over the intervals <math>[0,2]</math> and <math>[3,5]</math> compare to the average rates of change of <math>g(x) = 2^x</math> over the intervals <math>[0,2]</math> and <math>[3,5]</math>?</p> <p>12) Given a linear function and an exponential function, prove that linear functions grow by equal <i>differences</i> over equal intervals and that exponential functions grow by equal <i>factors</i> over equal intervals, by exploring both tables and graphs. Understand that a quantity increasing exponentially will eventually exceed a quantity increasing linearly.</p> <p>13) Compare the properties of two different functions when represented in different ways (i.e. a table, graph, or equation), including the domain, range, maximum, minimum, end behavior, and intervals where the function is increasing/decreasing.</p> <p>14) Solve linear equations in one variable algebraically, and</p>	<p>Example of a geometric sequence:  <math>3, 9, 27, 81, \dots</math>; which can be modeled by the exponential function <math>f(x) = 3^x</math> whose domain is integers greater than or equal to 1.</p> <p>Finding the average rate of change over a given interval of an exponential function (i.e. finding the “slope” between two points on a curve), for example:</p> <table border="1" data-bbox="1768 787 2091 971"> <thead> <tr> <th colspan="2"><math>f(x) = 2^x</math></th> </tr> <tr> <th><math>x</math></th> <th><math>F(x)</math></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>3</td> <td>8</td> </tr> </tbody> </table> <p>Average rate of range on the interval <math>[1,2]</math>:  <math>\frac{4 - 2}{2 - 1} = 2</math>                      Average rate of range on the interval <math>[2,3]</math>:  <math>\frac{8 - 4}{3 - 2} = 4</math>                      As the <math>x</math> values increase, the average rate of change also increases.</p> <p>Exploring the growth of linear</p>	$f(x) = 2^x$		$x$	$F(x)$	1	2	2	4	3	8		<p>11-13  <a href="http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/functions.pdf">http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/functions.pdf</a></p> <p>High School CCSS Flip Book  <a href="http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf">http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf</a></p>
$f(x) = 2^x$															
$x$	$F(x)$														
1	2														
2	4														
3	8														



Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources																
	<p>Learning experiences 14 – 15:  <a href="http://www.illustrativemathematics.org/illustrations/618">http://www.illustrativemathematics.org/illustrations/618</a></p>	<p>then represent the equivalent expressions from each side of the equal sign graphically or in a table of values. Assign each expression a function <math>f(x)</math> and <math>g(x)</math> and find the point of intersection for the input value of <math>x</math>.</p> <p>15) Solve equations exactly or approximately, where one or both sides of the equal sign are exponential expressions, both graphically and in a table, by assigning functions to each expression and finding the <math>x</math>-value of the point of intersection. Use technology to graph, create tables of values, or find successive approximations.</p>	<p>and exponential functions through tables:</p> <table border="1" data-bbox="1749 321 2107 472"> <thead> <tr> <th colspan="2"><math>f(x) = 2x</math></th> <th colspan="2"><math>g(x) = 2^x</math></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> <td>1</td> <td>2</td> </tr> <tr> <td>3</td> <td>6</td> <td>3</td> <td>8</td> </tr> <tr> <td>5</td> <td>10</td> <td>5</td> <td>32</td> </tr> </tbody> </table> <p><math>f(x)</math> is a linear function, and grows by an equal difference (+4) over equal intervals in the domain.</p> <p><math>g(x)</math> is an exponential function, and grows by an equal factor (<math>\times 2</math>) over equal intervals in the domain.</p> <p>Example of Experience 14:  <math>2x + 5 = 3x - 4</math>                      1) Solve algebraically. <math>x = 9</math>                      2) Represent each expression as separate functions, <math>f(x) = 2x + 5</math> and <math>g(x) = 3x - 4</math>                      3) Graph each function on the same coordinate plane and find the point of intersection                      4) Notice that the <math>x</math>-value of the point of intersection is the solution</p> <p>Example of Experience 15:</p>	$f(x) = 2x$		$g(x) = 2^x$		1	2	1	2	3	6	3	8	5	10	5	32		
$f(x) = 2x$		$g(x) = 2^x$																			
1	2	1	2																		
3	6	3	8																		
5	10	5	32																		

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
			$2^x = 5x + 7$ 1) Represent each expression as separate functions, $f(x) = 2^x$ and $g(x) = 5x + 7$ 2) By hand or by using technology, graph each function on the same coordinate plane or create a table of values, and approximate the x-value of the point of intersection of the two functions. 3) Test the solution by substituting it into the equation and check for equivalency and accuracy.		

Draft

## Unit #5: Building Functions

(Approx. # Days - 24)

Content Standards:

F.BF.1, 2, 3 and F.LE.1b, 1c, 2

### Math Common Core Content Standards:

#### Conceptual Category: Functions

#### Domain: Building Functions F-BF

**Build a function that models a relationship between two quantities.** [Linear and exponential functions (integer inputs).]

1. Write a function that describes a relationship between two quantities. ★
  - a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ★
  - b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* ★
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

#### Build new functions from existing functions.

3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* [In Mathematics I, this standard addresses linear and exponential functions and focuses on vertical translations for exponential functions.]

#### Domain: Linear, Quadratic, and Exponential Models F-LE

**Construct and compare linear, quadratic, and exponential models and solve problems.** [Linear and exponential functions.]

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. ★
  - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ★
  - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ★ [In Mathematics I, this standard addresses linear and exponential functions]
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★

#### Standards for Mathematical Practice:

1. Make Sense of Problems and Persevere in Solving Them
2. Reason Abstractly and Quantitatively
4. Model with Mathematics
5. Use Appropriate Tools Strategically
8. Look For and Express Regularity in Repeated Reasoning

<p><b>ELD Standards to Support Unit</b> [Add text]</p>	<p><b>SEL Competencies:</b> [Add text]</p>
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Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources												
<ul style="list-style-type: none"> <li>How do you use a sequence to write a function?</li> <li>How is the domain of a function different than the domain of a sequence?</li> <li>How do you know if you can write an explicit function (linear or exponential) for a given real-world math problem?</li> <li>How can you use a recursive formula for an arithmetic or geometric sequence to write an explicit formula?</li> <li>How can you determine whether a function is linear or exponential given a graph, verbal description, table, pattern, or recursive formula?</li> <li>What effect does a transformation of a graph of a function have on its equation? (For example, if the graph of a function is moved vertically upward 5 units, how has its equation changed?)</li> </ul>	<p>Assessments/Tasks aligned to learning experiences:</p> <p>For Learning Experience 10:  <a href="https://commoncorealgebra1.wikispaces.hcpss.org/file/view/F.BF.1+Maria%27s+Quinceanera.pdf">https://commoncorealgebra1.wikispaces.hcpss.org/file/view/F.BF.1+Maria%27s+Quinceanera.pdf</a></p>	<p>Students will be able to...</p> <ol style="list-style-type: none"> <li>Model a linear situation with a table and develop a recursive formula for an arithmetic sequence. Understand that recursion refers to building on the previous output value.</li> <li>Use a recursive formula to find other output values in an arithmetic sequence, for example if <math>p(n) = 5 + p(n - 1)</math> and <math>p(42) = 134</math>, find <math>p(41)</math> and <math>p(43)</math>.</li> <li>Given a context that models a linear function, build a table and a recursive formula for an arithmetic sequence, and write an “expanded pattern” to develop an explicit expression (i.e. linear function) to model the situation.*</li> <li>Model an exponential situation with a table and develop a recursive formula for a geometric sequence.</li> <li>Use a recursive formula to find other output values in a geometric sequence, for example if <math>p(n) = 2 \cdot p(n - 1)</math> and <math>p(7) = 100</math>, find <math>p(6)</math> and <math>p(8)</math>.</li> <li>Given a context that models an exponential function, build a table and a recursive formula for a geometric sequence, and write an “expanded pattern” to develop an explicit expression (i.e. exponential function) to model the situation.*</li> <li>In any given representation (for example a graph, verbal description, table, pattern or recursive formula, or explicit function) determine whether the relationship</li> </ol>	<p>Given a table of values, write a recursive formula, notice a pattern from the sequence, and write an explicit expression for a <b>linear</b> function:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>n</math></td> <td><math>f(n)</math></td> </tr> <tr> <td>0</td> <td>10</td> </tr> <tr> <td>1</td> <td>15</td> </tr> <tr> <td>2</td> <td>20</td> </tr> <tr> <td>3</td> <td>25</td> </tr> <tr> <td>4</td> <td>30</td> </tr> </table> <p><b>Recursive formula for sequence:</b> <math>f(n) = 5 + f(n - 1)</math> (Add 5 to the previous output from the table.)</p> <p><b>*Writing as an “expanded pattern”:</b>  <math>f(0) = 10</math>  <math>f(1) = 5 + 10</math> (or <math>5 + f(0)</math>)  <math>f(2) = 5 + 5 + 10</math> (or <math>5 + f(1)</math>)  <math>f(3) = 5 + 5 + 5 + 10</math> (or <math>5 + f(2)</math>)  <math>f(4) = 5 + 5 + 5 + 5 + 10</math> (or <math>5 + f(3)</math>)</p> <p><b>Explicit formula</b> (linear function): <math>f(n) = 5n + 10</math></p>	$n$	$f(n)$	0	10	1	15	2	20	3	25	4	30		<p><i>CA Mathematics Framework Math 1</i> p. 10 – 15  <a href="http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf">http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf</a></p> <p><i>Progressions for the Common Core – Gr. 8 and High School, Functions</i>  <a href="http://commoncoretools.me/wp-content/uploads/2013/07/css_progression_functions_2013_07_02.pdf">http://commoncoretools.me/wp-content/uploads/2013/07/css_progression_functions_2013_07_02.pdf</a></p> <p><i>Progressions for the Common Core – Modeling, High School</i>  <a href="http://commoncoretools.me/wp-content/uploads/2013/07/c">http://commoncoretools.me/wp-content/uploads/2013/07/c</a></p>
$n$	$f(n)$																
0	10																
1	15																
2	20																
3	25																
4	30																

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
		<p>can be modeled by a linear function or an exponential function.</p> <p>8) Solve real-world problems using exponential and linear functions.</p> <p>9) Explore relationships between two quantities that can be represented by a sequence and a recursive formula but <i>cannot</i> be modeled by a linear or exponential function, for example the Fibonacci sequence.</p> <p>10) Build a function that combines two different functions together, for example add or subtract an exponential function and a linear function to model a given situation.</p> <p>11) Discover the effects that a specific value <math>k</math> (positive or negative) has on the graph of <math>f(x)</math> of a linear function, including <math>f(x) + k</math>, <math>f(x + k)</math>, <math>kf(x)</math>, and <math>f(kx)</math>, by comparing the two functions through tables and graphs. (Framework p. 13)</p> <p>12) Discover the vertical effects that <math>f(x) + k</math> has on the graph of <math>f(x)</math> of an exponential function (where <math>k</math> is a specific positive or negative value), through tables and graphs.</p>	<p>(Follow the process above for writing exponential functions using geometric sequences).</p> <p>The Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, ...</p> <p>The Fibonacci sequence is recursive because in order to find any output within the sequence, you need to know the previous 2 outputs. The recursive formula is defined by:  <math>f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1)</math> for <math>n \geq 1</math>.</p> <p>Though this sequence is recursive, it is <i>not</i> an arithmetic or geometric sequence and can therefore not be modeled by a linear or exponential function.</p> <p>Compound interest formulas, such as <math>P(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt}</math> are often used to model real-world exponential situations.</p>		<p><a href="http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking-math/functions.pdf">css_progression_modeling_2013_07_04.pdf</a></p> <p>North Carolina Unpacked Content, HS Functions: pg.7-9, 11-13  <a href="http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking-math/functions.pdf">http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking-math/functions.pdf</a></p> <p><i>High School CCSS Flip Book</i>  <a href="http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf">http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf</a></p>

## Unit #6: Descriptive Statistics

(Approx. # Days - 20)

Content Standards: S.ID.1, 2, 3, 5, 6, 7, 8, 9

### Math Common Core Content Standards:

#### Conceptual Category: Statistics and Probability

#### Domain: Interpreting Categorical and Quantitative Data

##### Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ★
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

##### Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★
  - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. [Linear and exponential models only.] ★
  - b. Informally assess the fit of a function by plotting and analyzing residuals. ★
  - c. Fit a linear function for a scatter plot that suggests a linear association. ★

##### Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
8. Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
9. Distinguish between correlation and causation. ★

### Standards for Mathematical Practice:

1. Make Sense of Problems and Persevere in Solving Them
2. Reason Abstractly and Quantitatively
3. Construct Viable Arguments and Critique the Reasoning of Others
4. Model with Mathematics
5. Use Appropriate Tools Strategically
6. Attend to Precision
7. Look For and Make Use of Structure

<p><b>ELD Standards to Support Unit</b> [Add text]</p>	<p><b>SEL Competencies:</b> [Add text]</p>
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Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
<ul style="list-style-type: none"> <li>• How do you determine which model is best to represent a given data set?</li> <li>• What is an outlier?</li> <li>• How do outliers affect the mean and standard deviation?</li> <li>• Which graphical representations display each of the following: mean, median, interquartile range, and standard deviation?</li> <li>• How can you determine if the data is skewed? How will skewed data impact the mean, median, interquartile range, and standard deviation?</li> <li>• How do you use the different categories of the two-way frequency table to analyze the relationship between two variables?</li> <li>• What is the difference between a marginal frequency and a conditional relative frequency?</li> <li>• How can data be represented in order to promote a certain agenda?</li> </ul>	<p>Assessments/Tasks aligned to learning experiences:</p> <p><a href="http://www.illustrativemathematics.org/illustrations/942">http://www.illustrativemathematics.org/illustrations/942</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/1027">http://www.illustrativemathematics.org/illustrations/1027</a></p> <p><a href="http://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec_1_mod7_modeldata_tn_052313.pdf">http://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec_1_mod7_modeldata_tn_052313.pdf</a></p> <p>(This has multiple tasks that address the standards in this unit)</p> <p><a href="http://www.engageny.org/sites/default/files/resource/attachments/algebra_i-m2-teacher-materials.pdf">http://www.engageny.org/sites/default/files/resource/attachments/algebra_i-m2-teacher-materials.pdf</a></p> <p>(This module has a variety of lessons and tasks that pertain to this unit)</p>	<p>Students will be able to...</p> <ol style="list-style-type: none"> <li>1) Represent numerical data graphically using dot plots, histograms, and box plots, and analyze and interpret the data in terms of the situation it models. Analyze the strengths and weaknesses of each type of representation by comparing different plots of the same data.</li> <li>2) Understand the definitions of and calculate median, mean, interquartile range, and standard deviation and how they're represented for a given set of a data in a dot plot, histogram, and box plot. Understand that standard deviation represents the amount of variation from the mean in a given dataset.</li> <li>3) Given two sets of data or two graphs, identify the similarities and differences in shape, center (median, mean) and spread (interquartile range, standard deviation).</li> <li>4) Identify outliers and their effects on data sets for the purpose of determining an appropriate measure of center (median or mean) and spread (interquartile range or standard deviation) to describe a distribution that is symmetric or skewed. *</li> <li>5) Create a two-way frequency table from two categorical variables; read and interpret data and write clear summaries of data displayed in a two-way frequency table.</li> </ol>	<p>A statistical process is a problem-solving process consisting of four steps:</p> <ol style="list-style-type: none"> <li>1. Formulating a statistical question that anticipates variability and can be answered by data</li> <li>2. Designing and implementing a plan that collects appropriate data.</li> <li>3. Analyzing the data by graphical and/or numerical methods.</li> <li>4. Interpreting the analysis in the context of the original question.</li> </ol> <p>Recommend calculating standard deviation with smaller data sets for the purpose of understanding what standard deviation is and why outliers have a significant effect. The emphasis on standard deviation should be to understand its usefulness in</p>		<p><i>CA Mathematics Framework Math 1</i> p. 31 – 34 <a href="http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf">http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf</a></p> <p><i>Progressions for the Common Core – Gr. 8 and High School, Statistics and Probability</i> <a href="http://commoncoretools.org/wp-content/uploads/2012/06/css_progression_sphs_2012_04_21_bis.pdf">http://commoncoretools.org/wp-content/uploads/2012/06/css_progression_sphs_2012_04_21_bis.pdf</a></p> <p><i>Progressions for the Common Core – Modeling, High School</i> <a href="http://commoncoretools.org/wp-content/">http://commoncoretools.org/wp-content/</a></p>

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
<ul style="list-style-type: none"> <li>• How do you determine what kind of function should be used to fit a given data set?</li> <li>• How do you determine if the function models the data well?</li> <li>• What does a strong correlation mean?</li> <li>• What is the relationship between the residuals and the correlation coefficient?</li> <li>• What is the difference between correlation and causation?</li> </ul>	<p><a href="http://www.illustrativemathematics.org/illustrations/1307">http://www.illustrativemathematics.org/illustrations/1307</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/941">http://www.illustrativemathematics.org/illustrations/941</a></p> <p><a href="http://www.illustrativemathematics.org/illustrations/44">http://www.illustrativemathematics.org/illustrations/44</a></p>	<p>6) Understand and calculate joint, marginal, and conditional relative frequencies in a two way frequency table. Interpret the data in terms of the association between two variables by comparing conditional and marginal percentages.</p> <p>7) Create a scatter plot from two quantitative variables; identify the independent and dependent variables. Describe the correlation of the scatter plot in terms of its direction (positive, negative or none).</p> <p>8) Determine which form of a relationship (linear, exponential, or neither) should be used to represent a data set. Use technology to create a linear or exponential function to fit the data. Explain the meaning of the constant and coefficients of the function in context. Use the function to predict values.</p> <p>9) Understand how well the function fits the data by creating and analyzing a residual plot. Using technology, find and interpret the correlation coefficient and relate it to the residual plot.</p> <p>10) Understand that while the data and statistics may show a strong correlation, that is not always connected to causation. Distinguish between conditions of correlation and conditions of causation.</p>	<p>interpreting data.</p> <p>*Use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets to analyze data.</p> <p>Expose your students to multiple sources of data:</p> <p><a href="http://www.dartmouth.edu/~chance/teaching_aids/data.html">http://www.dartmouth.edu/~chance/teaching_aids/data.html</a></p> <p><a href="http://www.census.gov/#">http://www.census.gov/#</a></p> <p><a href="http://www.amstat.org/education/usefulsitesforteachers.cfm">http://www.amstat.org/education/usefulsitesforteachers.cfm</a></p> <p>Resources for all of these statistical concepts:</p> <p><a href="http://learnzillion.com/courses/50#collection_809">http://learnzillion.com/courses/50#collection_809</a></p> <p>Two-way frequency table</p> <p><a href="http://learnzillion.com/search?query=frequency&amp;page=1&amp;filters[common_core_code_s][]=S-ID.5">http://learnzillion.com/search?query=frequency&amp;page=1&amp;filters[common_core_code_s][]=S-ID.5</a></p> <p>Statistics Technology and</p>		<p><a href="https://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking-math/statistics-probability.pdf">uploads/2013/07/css_progression_modeling_2013_07_04.pdf</a></p> <p>North Carolina Unpacked Content, HS Statistics and Probability: pg. 2 – 6</p> <p><a href="http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking-math/statistics-probability.pdf">http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking-math/statistics-probability.pdf</a></p> <p><i>High School CCSS Flip Book</i></p> <p><a href="http://katm.org/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf">http://katm.org/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf</a></p>



Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
			<p>websites:  <a href="#">Use Excel for calculating statistical values</a>  <a href="http://www.alcula.com/calculators/statistics/">http://www.alcula.com/calculators/statistics/</a>  <a href="http://www.mathwarehouse.com/">http://www.mathwarehouse.com/</a></p> <p>The following website has a lot of lessons and tasks focused on statistics:  <a href="http://illuminations.nctm.org/Search.aspx?view=search&amp;st=d&amp;gr=9-12">http://illuminations.nctm.org/Search.aspx?view=search&amp;st=d&amp;gr=9-12</a></p>		

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## Unit #7: Congruence and Constructions

(Approx. # Days -26)

Content Standards: G.CO.1, 2, 3, 4, 5, 6, 7, 8, 12, 13

### Math Common Core Content Standards:

#### Conceptual Category: Geometry

#### Domain: Congruence

#### Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

#### Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

#### Make geometric constructions.

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

### Standards for Mathematical Practice:

1. Make Sense of Problems and Persevere in Solving Them
5. Use Appropriate Tools Strategically
6. Attend to Precision
7. Look For and Make Use of Structure

### ELD Standards to Support Unit

[Add text]

### SEL Competencies:

[Add text]

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
<ul style="list-style-type: none"> <li>• What are the similarities and differences between performing geometric constructions by hand and performing them using computer software?</li> <li>• Which transformations preserve distance and angle measure, and which do not? Why?</li> <li>• Does order always matter in a sequence of transformations? Why or why not?</li> <li>• Could there be more than one sequence of transformations that map one figure on to another? Explain with an example.</li> <li>• What is the definition of congruence in terms of rigid motion?</li> <li>• How can you determine if two figures are congruent?</li> <li>• Given two figures, how can you show that they are congruent (or not)?</li> <li>• Why are SSS, ASA, and SAS criteria for triangle congruence? How come SSA is <i>not</i> a criteria for triangle congruence?</li> </ul>	<p>Assessments/Tasks aligned to learning experiences:</p> <p>Learning Experiences 1 – 5:  <a href="http://www.illustrativemathematics.org/illustrations/1320">http://www.illustrativemathematics.org/illustrations/1320</a>  <a href="http://www.illustrativemathematics.org/illustrations/966">http://www.illustrativemathematics.org/illustrations/966</a>  <a href="http://www.illustrativemathematics.org/illustrations/1083">http://www.illustrativemathematics.org/illustrations/1083</a>  <a href="http://www.illustrativemathematics.org/illustrations/1557">http://www.illustrativemathematics.org/illustrations/1557</a></p> <p>Learning Experiences 5 – 8:  <a href="http://map.mathshell.org/materials/lessons.php?taskid=524#task524">http://map.mathshell.org/materials/lessons.php?taskid=524#task524</a>  <a href="http://www.illustrativemathematics.org/illustrations/1545">http://www.illustrativemathematics.org/illustrations/1545</a>  <a href="http://www.illustrativemathematics.org/illustrations/1509">http://www.illustrativemathematics.org/illustrations/1509</a>  <a href="http://www.illustrativemathematics.org/illustrations/1469">http://www.illustrativemathematics.org/illustrations/1469</a>  <a href="http://www.illustrativemathematics.org/illustrations/1471">http://www.illustrativemathematics.org/illustrations/1471</a></p> <p>Learning Experiences 8-9:</p>	<p>Students will be able to...</p> <ol style="list-style-type: none"> <li>1) Use the definitions of angle, circle, and line segment to perform the constructions: copy a line segment and bisect a line segment.*</li> <li>2) Use the definitions of angle, circle, and line segment to perform the constructions: copy an angle and bisect an angle.*</li> <li>3) Use the definitions of angle, circle, and line segment to perform the constructions: construct perpendicular lines, perpendicular bisector of a line segment, and construct a line parallel to a given line through a point not on the line.*</li> <li>4) Use previously learned constructions to inscribe an equilateral triangle, square and regular hexagon inside a circle.*</li> <li>5) Perform and describe a translation for a given figure and determine why the given translation preserves line segment distance, angle measure, and parallel and perpendicular relationships.</li> <li>6) Perform and describe a reflection for a given figure and determine why the given reflection preserves line segment distance, angle measure, and parallel and perpendicular relationships. Describe the lines of symmetry that reflect rectangles, parallelograms, trapezoids, and regular polygons onto themselves.</li> <li>7) Perform and describe a rotation for a given figure and determine why the given rotation preserves line segment distance, angle measure, and parallel and perpendicular relationships. Describe degrees of rotation that map rectangles, parallelograms,</li> </ol>	<p>*Use a variety of tools and methods to perform constructions.  <a href="#">GeoGebra</a> (online tool for dynamic geometry)  <a href="#">Euclid’s Elements and the definitions of angle, circle and line segment.</a>  <a href="#">Guide to basic constructions from Math is Fun</a>  <a href="#">Geometric Transformations using GeoGebra</a></p>		<p><i>CA Mathematics Framework Math 1</i>  p. 25 – 29  <a href="http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf">http://www.cde.ca.gov/ci/ma/cf/documents/aug2013mathematics1.pdf</a></p> <p>North Carolina Unpacked Content, HS Geometry: pg. 2-5, 8-9  <a href="http://www.ncpublicschools.org/docs/acre/standards/communication-core-tools/unpacking/math/geometry.pdf">http://www.ncpublicschools.org/docs/acre/standards/communication-core-tools/unpacking/math/geometry.pdf</a></p> <p><i>High School CCSS Flip Book</i>  <a href="http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf">http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf</a></p> <p>EngageNY  <a href="http://www.engageny.org/sites/default/">http://www.engageny.org/sites/default/</a></p>

Essential Questions	Suggested Assessments for Learning	Sequence of Learning Experiences	Strategies for Teaching and Learning	Differentiation (EL/SpEd/GATE)	Resources
	<p><a href="http://www.illustrativemathematics.org/illustrations/1546">http://www.illustrativemathematics.org/illustrations/1546</a></p> <p>For learning experiences 11 – 13:  <a href="http://www.illustrativemathematics.org/illustrations/31">http://www.illustrativemathematics.org/illustrations/31</a>  <a href="http://www.illustrativemathematics.org/illustrations/1547">http://www.illustrativemathematics.org/illustrations/1547</a>  <a href="http://www.illustrativemathematics.org/illustrations/1637">http://www.illustrativemathematics.org/illustrations/1637</a>  <a href="http://www.illustrativemathematics.org/illustrations/340">http://www.illustrativemathematics.org/illustrations/340</a>  <a href="http://map.mathshell.org/materials/lessons.php?taskid=452#task452">http://map.mathshell.org/materials/lessons.php?taskid=452#task452</a></p>	<p>trapezoids, and regular polygons onto themselves.</p> <p>8) Perform and describe a dilation for a given figure. Compare and contrast the effects of performing a dilation to a figure to the effects of performing translations, reflections, and rotations to a figure.</p> <p>9) Perform and describe a stretch for a given figure (e.g. a horizontal stretch or a vertical stretch). Compare and contrast the effects of stretching a figure to the effects of performing translations, reflections, and rotations to a figure.</p> <p>10) Specify a sequence of transformations (including rotations, reflections, translations, dilations, and/or stretches) that will carry a given figure onto another.</p> <p>11) Use a sequence of transformations to transform figures, and predict if two figures are congruent based on the sequence of transformations.</p> <p>12) Use rigid motions to show that congruent triangles have congruent corresponding parts (sides and angles). Conversely, show that two triangles are congruent by describing a sequence of transformations that takes one triangle on to the other. Make conjectures about possible criteria for triangle congruence.</p> <p>13) Use the definition of congruence in terms of rigid motions to understand and explain the criteria for triangle congruence (ASA, SAS, and SSS).</p>	<p>Definition of congruence in terms of rigid motions:  <i>Two shapes are congruent if there is a sequence of rigid motions in the plane that takes one shape exactly onto the other</i> (Framework, p. 26).</p> <p><i>Note:</i> In standards G-CO.1-8, formal proof is <i>not</i> required. Students are asked to show using transformations that certain results are true.</p>		<p><a href="#">files/resource/attachments/geometry-m1-teacher-materials.pdf</a></p> <p><a href="#">Massachusetts DOE – Transformations Unit of Study</a></p>

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