# Unit of Study
## Ratio Relationships

### Grade: 6  
**Topic:** Ratios and Unit Rates  
**Length of Unit:** 20 – 25 days

### Focus of Learning

#### Common Core Standards:

**Understand ratios concepts and use ratio reasoning to solve problems.**

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.RP.1</td>
<td>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</td>
</tr>
<tr>
<td>6.RP.2</td>
<td>Understand the concept of a unit rate (\frac{a}{b}) associated with a ratio (a:b) with (b\neq 0), and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is (\frac{3}{4}) cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”</td>
</tr>
</tbody>
</table>
| 6.RP.3   | Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.  
  a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.  
  b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?  
  c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.  
  d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. |

#### Mathematical Practices:

1. Make sense of problems and persevere in solving them.  
2. Reason abstractly and quantitatively.  
3. Construct viable arguments and critique the reasoning of others.  
4. Model with mathematics.  
5. Use appropriate tools strategically.  
6. Attend to precision.  
7. Look for and make use of structure.  
8. Look for and express regularity in repeated reasoning.

### Enduring Understanding(s):

Students will understand that...

1. A ratio or a rate expresses the relationship between two quantities.  
2. Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems.  
3. A ratio is a distinct entity, different from the two measures that make it up.

### Guiding Questions: These questions will guide student inquiry.

1. Why are ratios important?  
2. How are ratios used in everyday life?  
3. What kind of problems can I solve with ratios?  
4. When is it useful to be able to relate one quantity to another?  
5. How can I compare two different quantities?  
6. How are ratios and rates similar and different?

### Student Performance

#### Knowledge: Students will understand/know...

- A ratio compares two related quantities  
- Ratios can be represented in multiple formats including for each, per, to, each, %, 1:5, 1/5, 0.2, etc.  
- Strategies for solving ratio problems  
- Appropriate use of mathematical strategies for solving problems involving ratios and rates such as tables of equivalent ratios, tape diagrams, double number lines, graphs or equations  
- A rate is a special kind of ratio that compares two types of measurement

#### Application: Students will be able to...

- Use ratio and rate language  
- Write ratios to describe the relationship between two quantities  
- Use tables to compare ratios  
- Make and manipulate tables of equivalent ratios  
- Plot pairs of values on the coordinate plane  
- Use double number lines to solve problems  
- Use tape diagrams to solve problems  
- Propose, justify and communicate solutions  
- Find unit rates using tools such as tables, tape diagrams
- A unit rate is the ratio of two measurements in which the second term is 1
- A percent is a type of ratio that compares a quantity to 100
- The quantity represented by a percent depends upon the size of the whole
- Percent problems contain three components; the percent, the part and the whole.

### Assessments (Attached)

**Pre-Assessment:**

**Formative Interim Assessment (Mid-Unit Checks):**
- MARS – 6th grade 2002 “Grandpa’s Knitting” (Lesson 6)
- MARS – 6th grade 2001 “Cans of Kola” (Lesson 10)

**Suggested Formative Assessments:**
- MARS – 7th grade 2006 “Square Tiles” (Use after Lesson 2)
- SBAC-MAT.06.CR.1.000RP.A.174 (Use after Lesson 2)
- Illustrative Mathematics-6.RP.A.3 “Mixing Concrete” (Use after Lesson 5)
- Illustrative Mathematics-6.RP.3 “Friends Meeting on Bikes” (Use after Lesson 9)
- Illustrative Mathematics-6.RP.A.3.c “Shirt Sale” (Use after Lesson 12)

**Post-Assessment (Culminating Task):**
- SBAC – MAT.06.PT.4.BDBRC.A.280 Claim 4 “Bead Bracelet” (Lesson 15)

### Learning Experiences (Lesson Plans Attached)

<table>
<thead>
<tr>
<th>Days</th>
<th>Lesson Sequence</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Lesson 1: Introduction to Ratios</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>Students will know:</em></td>
<td>Suggested Formative Assessments:</td>
</tr>
<tr>
<td></td>
<td>• A ratio compares two related quantities</td>
<td>• MARS – 7th grade 2006 “Square</td>
</tr>
<tr>
<td></td>
<td>• Ratios can be represented in multiple formats including <em>for each, per, to, each, %, 1/5, etc.</em></td>
<td>Tiles”</td>
</tr>
<tr>
<td></td>
<td><em>Students will be able to:</em></td>
<td>• SBAC-MAT.06.CR.1.000RP.A.174</td>
</tr>
<tr>
<td></td>
<td>• Use ratio and rate language to describe the relationship between two quantities</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Lesson 2: Writing Ratios</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>Students will know:</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• A ratio compares two related quantities</td>
<td>Suggested Formative Assessments:</td>
</tr>
<tr>
<td></td>
<td>• Ratios can be represented in multiple formats including <em>for each, per, to, each, %, 1:5, 1/5, etc.</em></td>
<td>• MARS – 7th grade 2006 “Square</td>
</tr>
<tr>
<td></td>
<td><em>Students will be able to:</em></td>
<td>Tiles”</td>
</tr>
<tr>
<td></td>
<td>• Write ratios to describe the relationship between two quantities</td>
<td>• SBAC-MAT.06.CR.1.000RP.A.174</td>
</tr>
<tr>
<td></td>
<td><strong>Lesson 3: Problem Solving with Ratios (tables of equivalent ratios)</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>Students will know:</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Appropriate use of mathematical strategies for solving problems involving ratios and rates such as tables of equivalent ratios, tape diagrams, double number lines, graphs or equations</td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>Students will be able to:</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Use tables to compare ratios</td>
<td></td>
</tr>
<tr>
<td>Lesson 4: Problem Solving with Ratios (double number lines)</td>
<td>Students will know:</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------------------</td>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Make and manipulate tables of equivalent ratios</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Plot pairs of values on the coordinate plane</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Students will be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use double number lines to solve problems</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson 5: Problem Solving with Ratios (tape diagrams)</th>
<th>Students will know:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Appropriate use of mathematical strategies for solving problems involving ratios and rates such as tables of equivalent ratios, tape diagrams, double number lines, graphs or equations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Students will be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use tape diagrams to solve problems</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Suggested Formative Assessment:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illustrative Mathematics- 6.RP.A.3 “Mixing Concrete”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson 6: Ratios - Review and Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will:</td>
</tr>
<tr>
<td>Propose, justify and communicate solutions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interim Assessment:</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARS – 6th grade 2002 “Grandpa’s Knitting”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson 7: Understanding Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will know:</td>
</tr>
<tr>
<td>A rate is a special kind of ratio that compares two types of measurement</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Students will be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find rates using tools such as tables, tape diagrams and double number lines</td>
</tr>
<tr>
<td>Use rate language</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson 8: Understanding Unit Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will know:</td>
</tr>
<tr>
<td>A unit rate is the ratio of two measurements in which the second term is 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Students will be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find unit rates using tools such as tables, tape diagrams and double number lines</td>
</tr>
<tr>
<td>Use rate language</td>
</tr>
<tr>
<td>Convert measurement units</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson 9: Solve Problems with Rates and Unit Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will know:</td>
</tr>
<tr>
<td>A unit rate is the ratio of two measurements in which the second term is 1</td>
</tr>
<tr>
<td>Appropriate use of mathematical strategies for solving problems involving ratios and rates such as tables of equivalent ratios, tape diagrams, double number lines, graphs or equations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Students will be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find unit rate and use to solve problems including those with unit pricing and constant speed</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Suggested Formative Assessment:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illustrative Mathematics-6.RP.3 “Friends Meeting on Bikes”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson 10: Rates and Unit Rates - Review and Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will:</td>
</tr>
<tr>
<td>Propose, justify and communicate solutions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interim Assessment:</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARS – 6th grade 2001 “Cans of Kola” (Note: #3 may not apply)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson 11: Understanding Percents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will know:</td>
</tr>
<tr>
<td>A percent is a type of ratio that compares a quantity to 100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Students will be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe ratios as percents</td>
</tr>
</tbody>
</table>
### Lesson 12: Visual Representations of Percents

**Students will know:**
- A percent is a type of ratio that compares a quantity to 100

**Students will be able to:**
- Use tape diagrams to solve problems
- Use double number lines to solve problems

**Suggested Formative Assessment:**
- Illustrative Mathematics-6.RP.A.3.c “Shirt Sale”

### Lesson 13: Solving Problems with Percents

**Students will know:**
- The quantity represented by a percent depends upon the size of the whole
- Percent problems contain three components; the percent, the part and the whole.

**Students will be able to:**
- Represent a percent of a number
- Write a statement in the form of ___% of ___ = ___
- Solve problems where they must find the unknown part or percent (given the other two values)

### Lesson 14: Review

**Students will:**
- Propose, justify and communicate solutions

### Lesson 15: Culminating Task

**Students will:**
- Show their knowledge and understanding of ratios and unit rates.

**Summative Assessment:**
- SBAC--MAT.06 PT.4. BDBRC, A.280 Claim 4 “Bead Bracelet”

### Resources

#### Online

- Georgia Department of Education
  [https://www.georgiastandards.org/CommonCore/Pages/Math.aspx](https://www.georgiastandards.org/CommonCore/Pages/Math.aspx)

- Illustrative Mathematics

- Inside Mathematics/MARS tasks

- Massachusetts Department of Elementary and Secondary Education
  [http://www.doe.mass.edu/candi/model/units/Mathg6-RatioRates.docx](http://www.doe.mass.edu/candi/model/units/Mathg6-RatioRates.docx)

- National Library of Virtual Manipulatives

- North Carolina Department of Public Instruction
  [http://www.dpi.state.nc.us/acre/standards/common-core-tools/#unmath](http://www.dpi.state.nc.us/acre/standards/common-core-tools/#unmath)

- Progressions for the Common Core State Standards in Mathematics
  [http://ime.math.arizona.edu/progressions/](http://ime.math.arizona.edu/progressions/)

#### Text


Smarter Balanced Assessment Consortium
http://www.smarterbalanced.org/smarter-balanced-assessments/#item

Utah State Office of Education
Grandpa's Knitting

This problem gives you the chance to:
- apply measures in a practical situation

John's grandfather enjoys knitting.

He can knit a scarf 30 inches long in 10 hours.
He always knits for 2 hours each day.

1. How many inches can he knit in 1 hour? _______ inches
Show your calculations.

2. How many days will it take Grandpa to knit a scarf 30 inches long? _______ days
Show your calculations.

3. How many inches long will the scarf be at the end of 2 days? _______ inches
Explain how you figured it out.
4. How many hours will it take Grandpa to knit a scarf 27 inches long?  

____________________ hours  

Explain your reasoning.  

______________________________________________________________________________  

______________________________________________________________________________  

______________________________________________________________________________  

5. Grandpa uses $17\frac{1}{2}$ ounces of wool to make 5 scarves.  

How many ounces of wool will he need for 3 scarves?  

____________________ ounces  

Explain how you figured it out.  

______________________________________________________________________________  

______________________________________________________________________________  

______________________________________________________________________________
<table>
<thead>
<tr>
<th>Grandpa's Knitting</th>
<th>Test 6 Form A Rubric</th>
</tr>
</thead>
<tbody>
<tr>
<td>The core elements of performance required by this task are:</td>
<td></td>
</tr>
<tr>
<td>• apply measures in a practical situation</td>
<td></td>
</tr>
<tr>
<td>Based on these, credit for specific aspects of performance should be assigned as follows:</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Points</th>
<th>Section Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

1. Gives correct answer as:
   3 inches and shows $30 \div 10$ or equivalent.

2. Gives correct answer as:
   5 days and shows $10 \div 2$ or equivalent.

3. Gives correct answer as:
   12 inches
   Gives correct explanation such as:
   In 1 day he knits $3 \times 2 = 6$ inches.
   In 2 days he knits $2 \times 6$ inches.

   *Allow partial credit for a partially correct explanation.*

4. Gives correct answer as:
   9 hours
   Gives correct explanation such as:
   To knit 27 inches takes $27 \div 3$ hours.

5. Gives correct answer as:
   $10\frac{1}{2}$ ounces
   Gives correct explanation such as:
   1 scarf takes $17\frac{1}{2} + 5 = 3\frac{1}{2}$ ounces.
   3 scarves take $3 \times 3\frac{1}{2}$ ounces.

   *Allow partial credit for a partially correct explanation.*

Total Points | 10 |
Cans of Kola

This problem gives you the chance to:
• use numbers in a practical situation

Chris wanted to see which Kola Kola was the best price.

12-pack $3.20

6-pack $1.60

24-pack $4.24

She wrote:

The 6-pack costs $1.60

The 12-pack costs $3.20, so I divided by 2 on the calculator.

The 24-pack costs $4.24, so I divided by 4 on the calculator.

To find the best price, I just compare these three values.
1. Will Chris’s method of figuring out the best price for soda work? Explain your reasoning.

2. Chris said, “The 6-pack costs $1.60. The calculator shows that 6 cans from the 12-pack cost 1.6 and 6 cans from the 24-pack cost 1.06. I’m not sure which is the best price.”

Write a short note to Chris to help her understand the meaning of 1.60, 1.6 and 1.06. Then recommend the “best buy” to her, explaining how you know it is the lowest price.

3. Chris has one coupon for 10% off any Kola Kola purchases. Find the lowest price Chris would pay for 30 cans. Explain how you got your answer.
<table>
<thead>
<tr>
<th></th>
<th>Cans of Kola: Grade 6</th>
<th>points</th>
<th>section points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gives correct answer such as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes. She is finding the cost of 6 cans from each of the packs.</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Gives correct explanation such as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.60 means 1 dollar and 60 cents</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.6 means 1 dollar and 6 tenths of a dollar = 1 dollar and 60 cents</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.06 means 1 dollar and 6 hundredths of a dollar = 1 dollar and 6 cents.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6 cans from the 24 pack are cheapest/ the best buy.</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Gives correct explanation such as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>The cheapest cost of 24 cans = $4.24</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>With 10% off the cost of 24 cans = $3.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Six cans cost $1.60.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>With 10% off the cost of 6 cans = $1.44</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Total cost of 30 cans = $5.26</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>or</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>The student may reduce the total price by 10%:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>The cheapest cost of 30 cans /24 pack + 6 pack = $5.84</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Finds 10% of the cheapest cost of 30 cans</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Finds that the cheapest cost with 10% off = $5.26</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Total Points</td>
<td></td>
<td>8</td>
</tr>
</tbody>
</table>
### MAT.06.PT.4.BDBRC.A.280 Claim 4

<table>
<thead>
<tr>
<th>Sample Item ID:</th>
<th>MAT.06.PT.4.BDBRC.A.280</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title:</td>
<td>Bead Bracelet (BDBRC)</td>
</tr>
<tr>
<td>Grade:</td>
<td>06</td>
</tr>
</tbody>
</table>
| Primary Claim: | **Claim 4: Modeling and Data Analysis**  
Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems. |
| Secondary Claim(s): | **Claim 1: Concepts and Procedures**  
Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. |
| Primary Content Domain: | Ratios and Proportional Relationships |
| Secondary Content Domain(s): | Equations and Expressions, The Number System, Numbers and Operations in Base Ten |
| Assessment Target(s): | 4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.  
4 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.  
4 D: Interpret results in the context of a situation.  
1A: Understand ratio concepts and use ratio reasoning to solve problems.  
1F: Reason about and solve one-variable equations and inequalities.  
1 G: Represent and analyze quantitative relationships between dependent and independent variables.  
1 C: Compute fluently with multi-digit numbers and find common factors and multiples.  
1 C (Gr 5): Understand the place-value system. |
| Standard(s): | 6.RP.1, 6.RP.2, 6.RP.3, 6.EE.7, 6.EE.9, 6.NS.3, 5.NBT.4 |
| Mathematical Practice(s): | 1, 3, 4, 5 |
| DOK: | 3 |
| Item Type: | PT |
| Score Points: | 16 |
| Difficulty: | H |

**How This Task Addresses The “Sufficient Evidence” For This Claim:**  
The student carries out mathematical procedures with precision when determining the design of a bracelet. Once the design is determined, the student uses ratio and proportion to determine the number and type of beads needed for a necklace, as well as uses properties of inequalities in some instances. Finally, the student creates a cost analysis by determining the cost of the bracelet and necklace, along with the profit for the items when given a certain percentage.
Bead Bracelets

Your school is hosting an Arts and Crafts Fair to raise funds. Your class has been asked to help by designing and making jewelry for the fund-raiser. In this task, you will be asked to design a bracelet, calculate ratios, make predictions, and calculate costs.

Designing a Bracelet

Part A

Your principal has purchased the materials to make the jewelry. The materials include:

- Three types of glass beads
- Three types of spacer beads (the beads used to separate sections of glass beads)
- Beading wire (the wire that holds the beads when making
a bracelet or a necklace)

- Clasps (the fasteners that hold the ends of a bracelet or necklace together)

The cost of each type of bead is shown below.

### Glass Beads

<table>
<thead>
<tr>
<th>Type</th>
<th>Cost for a bag of beads</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$4.25 for a bag of 48 beads</td>
</tr>
<tr>
<td>B</td>
<td>$6.00 for a bag of 25 beads</td>
</tr>
<tr>
<td>C</td>
<td>$8.00 for a bag of 25 beads</td>
</tr>
</tbody>
</table>

### Spacer Beads

<table>
<thead>
<tr>
<th>Type</th>
<th>Cost for a bag of beads</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>$4.00 for a bag of 25 beads</td>
</tr>
<tr>
<td>E</td>
<td>$8.00 for a bag of 24 beads</td>
</tr>
<tr>
<td>F</td>
<td>$7.00 for a bag of 300 beads</td>
</tr>
</tbody>
</table>
Design a bracelet using at least **two** types of glass beads and **one** type of spacer bead.

- Use between 8 and 12 glass beads.
- Use at least 6 spacer beads.
- Use no more than 25 total beads in your bracelet.

Write the type letter (A, B, C, D, E, or F) to represent each bead in your design. Use the 25 blanks below to lay out the design for your bracelet. Only write one letter in each blank you use.

__ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Write 5 ratios that can be used to mathematically describe the bracelet you designed. Make sure your ratios show each of the following:

- The relationship between one type of glass bead used and another type of glass bead used
- The relationship between one type of glass bead used and all the beads used
- The relationship between one type of glass bead used and a type of spacer bead used
- The relationship between all the glass beads used and all the spacer beads used
- The relationship between one type of spacer bead used and all the beads used
You have been given one bag of each type of bead that you have selected. Based on your design, how many complete bracelets can you make before you run out of one type of bead? Explain your answer using diagrams, mathematical expressions, and/or words.

**Part B**

**Calculating the Costs**

The cost of one clasp and enough beading wire to make a bracelet is $0.25. Using the information from *Part A*, determine the cost to create one of the bracelets you designed. Explain your answer using diagrams, mathematical expressions, and/or words.

In *Part A*, you determined the number of complete bracelets you could make before running out of one type of bead. Determine the cost to create this number of bracelets. Explain your answer using diagrams, mathematical expressions, and/or words.
Part C

Matching Necklaces

Your principal would like you to make some necklaces to match the bracelets you designed.

- The cost of one clasp and enough beading wire to make a 24-inch necklace is $0.30.
- Your bracelet is 8 inches long.

Determine the cost to create a 24-inch necklace that contains the same ratios of beads as your bracelet contains. Explain your answer using diagrams, mathematical expressions, and/or words.

Approximately how many of each type of bead will be needed to create a 24-inch necklace? Explain your answer using diagrams, pictures, mathematical expressions, and/or words.
Part D

Predicting Profits

[The teacher should discuss the definition of profit in this context. “A profit is the amount of money that is earned when a product is sold. Profit is determined by subtracting the cost of making the products from the price charged to customers.”]

For the Arts and Crafts Fair, your principal sets the price of each bracelet and necklace such that the school makes a profit that is 60% of the cost to make each piece of jewelry.

Determine the price at which your bracelet and necklace will be sold at the Arts and Crafts Fair. Explain your answer using diagrams, pictures, mathematical expressions, and/or words.

Your principal would also like to offer discounted prices for customers who buy sets of 3 bracelets. When customers buy sets of 3 bracelets, the school will make a profit that is 40% of the cost to make each bracelet. Determine the price at which a set of 3 bracelets will be sold at the Arts and Crafts Fair. Explain
The list below shows the pieces of jewelry that were sold at the Arts and Crafts Fair.

- 5 sets of 3 bracelets
- 4 necklaces
- 20 individual bracelets

Determine the **total** profit the school made from selling these pieces of jewelry. Explain your answer using diagrams, mathematical expressions, and/or words.
Sample Top-Score Response:

Part A

F, D, A, D, A, D, F, E, D, A, D, A, D, F, E, D, A, D, A, D, F

(highlighted for visual)

Ratios will vary based upon the layout of beads chosen by the student.

1 Type B glass bead to 3 Type A glass beads (1:3)
3 Type A glass beads to 1 Type B glass bead (3:1)

6 Type A glass beads out of 23 beads in total (6:23)
2 Type B glass beads out of 23 beads in total (2:23)

2 Type A glass beads to 3 Type D spacer beads (2:3)
1 Type A glass bead to 1 Type F spacer bead (1:1)
2 Type B glass beads to 9 Type D spacer beads (2:9)
2 Type B glass beads to 6 Type F spacer beads (1:3)

8 glass beads to 15 spacer beads (8:15)
9 Type D spacer beads out of 23 beads in total (9:23)
6 Type F spacer beads out of 23 beads in total (6:23)

I can make 2 bracelets. There are only 25 Type D spacer beads in a package, and my bracelet used 9 per bracelet. \(25 \div 9 = 2 \text{ R} 7\), so I can only make 2 complete bracelets before I run out of Type D spacer beads.

Part B

\[\begin{align*}
4.25 \div 48 &= 0.089 \text{ so } $0.09 \text{ per Type A glass bead} \\
6.00 \div 25 &= 0.24 \text{ so } $0.24 \text{ per Type B glass bead} \\
4.00 \div 25 &= 0.16 \text{ so } $0.16 \text{ per Type D spacer bead} \\
7.00 \div 300 &= 0.023 \text{ so } $0.02 \text{ per Type F spacer bead} \\
6($0.09) + 2($0.24) + 9($0.16) + 6($0.02) &= $2.83 \\
2($2.83) &= $5.66
\end{align*}\]

Part C

\[\begin{align*}
$2.83 - $0.25 &= $2.58; $2.58 \times 3 + $0.30 &= $8.04
\end{align*}\]

The 8-inch bracelet was designed with 6 Type A glass beads. Based on this design, a 24-inch necklace would have 18 of these beads.

There are 2 Type B glass beads in the 8-inch bracelet. The 24-inch necklace would have 6 of these beads.
There are 9 Type D spacer beads in the 8-inch bracelet. The 24-inch necklace would have 27 of these beads.

There are 6 Type F spacer beads in the 8-inch bracelet. The 24-inch necklace would have 18 of these beads.

OR

\[ 23 \div 8 = 2.875 \text{ beads per inch} \]
\[ 2.875 \times 24 = 69 \text{ beads on a 24-inch necklace} \]

\[ 23 \div 6 = 3.83 \]
\[ 69 \div 3.83 = 18.02 \]

There will be approximately 18 Type A glass beads and 18 Type F spacer beads on the necklace.

\[ 23 \div 2 = 11.5 \]
\[ 69 \div 11.5 = 6 \]

There will be approximately 6 Type B glass beads on the necklace.

\[ 23 \div 9 = 2.56 \]
\[ 69 \div 2.56 = 26.95 \]

There will be approximately 27 Type D spacer beads on the necklace.

OR

\[ \frac{6}{23} = \frac{n}{69} \]
\[ 6(69) = 23n \]
\[ 414 = 23n \]
\[ 414 \div 23 = n \]
\[ 18 = n \]

There will be approximately 18 Type A glass beads and 18 Type F spacer beads on the necklace.

\[ \frac{2}{23} = \frac{n}{69} \]
\[ 2(69) = 23n \]
\[ 138 = 23n \]
\[ 138 \div 23 = n \]
\[ 6 = n \]

There will be approximately 6 Type B glass beads on the necklace.
Grade 6 Mathematics Sample PT Form Claim 4

<table>
<thead>
<tr>
<th>9</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>69</td>
</tr>
</tbody>
</table>

\[ 9(69) = 23n \]
\[ 621 = 23n \]
\[ 621 \div 23 = n \]
\[ 27 = n \]

There will be approximately 27 Type D spacer beads on the necklace.

**Part D**

\[ \$2.83 \times 1.6 = \$4.53 \]
\[ \$8.04 \times 1.6 = \$12.86 \]

\[ (\$2.83 \times 3) \times 1.4 = \$11.89 \]

Profit from sets of bracelets:
\[ \$11.89 \times 5 = \$59.45; \$2.83 \times 15 = \$42.45; \$59.45 - \$42.45 = \$17.00 \]

Profit from necklaces:
\[ \$12.86 \times 4 = \$51.44; \$8.04 \times 4 = \$32.16; \$51.44 - \$32.16 = \$19.28 \]

Profit from individual bracelets:
\[ \$4.53 \times 20 = \$90.60; \$2.83 \times 20 = \$56.60; \$90.60 - \$56.60 = \$34.00 \]

Total profit:
\[ \$17.00 + \$19.28 + \$34.00 = \$70.28 \]

**Scoring Notes:**
Each section is evaluated independently. The total number of points is determined by adding the points assigned for each task.

**Scoring Rubric:**

**Part A**

**6 points:** Thorough understanding of ratio and proportional relationships. Thorough understanding of the given directions. The student correctly used one type of spacer bead and at least two types of glass beads. The student correctly used no more than 25 total beads and correctly used 8 to 12 glass beads and at least 6 spacer beads. The student correctly wrote a set of 5 ratios according to bulleted directions. The student correctly used mathematics to find the number of bracelets that can be made using all the different types of beads the student chose for the bracelet.

**5 points:** Thorough understanding of ratio and proportional relationships. Partial understanding of the given directions. The student correctly used one type of spacer bead and at least two types of glass beads. The student used a number of glass beads or spacer beads that were outside of directions. The student correctly wrote a set of 5 ratios according to bulleted directions. The student correctly used mathematics to find the number of bracelets that can be made using all the different types of beads the student chose for the bracelet. OR The student did everything else required, but only correctly wrote 4 of the 5...
required ratios. OR The student did everything else required, but did not correctly determine the number of bracelets that could be made.

4 points: Partial understanding of ratio and proportional relationships. Partial understanding of the given directions. The student did everything else required, but only correctly wrote 3 of the 5 required ratios. OR The student did everything else required, but only correctly wrote 4 of the 5 required ratios and did not correctly determine the number of bracelets that could be made. OR The student did everything else required, but used a number of glass beads or spacer beads that were outside of directions and only correctly wrote 4 of the 5 required ratios. OR The student did everything else required, but used a number of glass beads or spacer beads that were outside of directions and did not correctly determine the number of bracelets that could be made.

3 points: Partial understanding of ratio and proportional relationships. Partial understanding of the given directions. The student did everything else required, but only correctly wrote 2 of the 5 required ratios. OR The student did everything else required, but only correctly wrote 3 of the 5 required ratios and did not correctly determine the number of bracelets that could be made. OR The student did everything else required, but used a number of glass beads or spacer beads that were outside of directions and only correctly wrote 3 of the 5 required ratios. OR The student used a number of glass beads or spacer beads that were outside of directions, made an error with 1 ratio, and did not correctly determine the number of bracelets that could be made.

2 points: Partial understanding of ratio and proportional relationships. Partial understanding of the given directions. The student did everything else required, but only correctly wrote 1 of the 5 required ratios. OR The student did everything else required, but only correctly wrote 2 of the 5 required ratios and did not correctly determine the number of bracelets that could be made. OR The student did everything else required, but used a number of glass beads or spacer beads that were outside of directions and only correctly wrote 2 of the 5 required ratios. OR The student used a number of glass beads or spacer beads that were outside of directions, made an error with 2 ratios, and did not correctly determine the number of bracelets that could be made.

1 point: Limited understanding of ratio and proportional relationships. Limited understanding of the given directions. The student used a number of glass beads or spacer beads that were outside of directions, made an error with 3 or more ratios, and did not correctly determine the number of bracelets that could be made. OR The student used a number of glass beads or spacer beads that were outside of directions, made an error with 4 or 5 ratios, but correctly determined the number of bracelets that could be made.

0 points: No understanding of ratio and proportional relationships. No understanding of the given directions. The student made errors in every section of Part A.

Part B

3 points: Thorough understanding of numbers and operations. Thorough understanding of solving real-world problems involving the cost of making bracelets. The student correctly determines the minimum cost of the bracelet by first dividing the total cost of each package of beads by the number of beads in the package. Then the student correctly multiplies each individual cost by the number of each type of bead in the bracelet. The student correctly determines the cost of the total number of bracelets created from one bag of each style of
bead by multiplying the number of bracelets that can be made and the cost of each individual bracelet.

2 points: Partial understanding of numbers and operations. Partial understanding of solving real-world problems involving the cost of making bracelets. The student correctly determines the minimum cost of the bracelet by first dividing the total cost of each package of beads by the number of beads in the package. Then the student correctly multiplies each individual cost by the number of each type of bead in the bracelet. The student incorrectly determines the cost of the total number of bracelets created from one bag of each style of bead when multiplying the number of bracelets that can be made and the cost of each individual bracelet.

1 point: Limited understanding of numbers and operations. Limited understanding of solving real-world problems involving the cost of making bracelets. The student correctly determines the minimum cost of the bracelet by first dividing the total cost of each package of beads by the number of beads in the package. Then the student incorrectly multiplies each individual cost by the number of each type of bead in the bracelet. The student incorrectly determines the cost of the total number of bracelets created from one bag of each style of bead when multiplying the number of bracelets that can be made and the cost of each individual bracelet.

0 points: No understanding of numbers and operations. No understanding of solving real-world problems involving the cost of making bracelets. The student incorrectly determines the minimum cost of the bracelet when dividing the total cost of each package of beads by the number of beads in the package. Then the student incorrectly multiplies each individual cost by the number of each type of bead in the bracelet. The student incorrectly determines the cost of the total number of bracelets created from one bag of each style of bead when multiplying the number of bracelets that can be made and the cost of each individual bracelet.

Part C

4 points: Through understanding of ratio and proportions. Thorough understanding of mathematical expressions. The student correctly determines the cost for each inch of the necklace by subtracting $0.25, multiplying the cost of the bracelet by 3, and adding $0.30. The student correctly determines the number of each type of bead that would be needed for the necklace.

3 Points: Partial understanding of ratio and proportions. Partial understanding of mathematical expressions. The student correctly determines the cost for each inch of the necklace by subtracting $0.25, multiplying the cost of the bracelet by 3, and adding $0.30. The student makes an error when determining the number of 1 type of bead that would be needed for the necklace. OR The student makes an error when determining the cost of the necklace, but correctly determines the number of each type of bead that would be needed for the necklace.

2 points: Partial understanding of ratio and proportions. Partial understanding of mathematical expressions. The student correctly determines the cost for each inch of the necklace by subtracting $0.25, multiplying the cost of the bracelet by 3, and adding $0.30. The student makes an error when determining the number of 2 types of bead that would be needed for the necklace. OR The student makes an error when determining the cost of the necklace and makes an error when determining the number of 1 type of bead that would be
needed for the necklace.

1 point: Limited understanding of ratio and proportions. Limited understanding of mathematical expressions. The student correctly determines the cost for each inch of the necklace by subtracting $0.25, multiplying the cost of the bracelet by 3, and adding $0.30. The student does make errors in determining the number of 3 or more of the bead types needed to make the necklace. OR The student makes an error when determining the cost of the necklace and makes an error when determining the number of 2 types of bead that would be needed for the necklace.

0 points: No understanding of ratio and proportions. No understanding of mathematical expressions and inequalities. The student does not correctly complete any section of Part C.

Part D

3 points: Thorough understanding of numbers and operations and the number system. The student correctly determines the profit of 60% by multiplying the cost of the bracelet by 1.6 and the cost of the necklace by 1.6. The student correctly determines the 40% profit from selling a set of 3 bracelets by multiplying the cost of the bracelet by 3 and then multiplying that total by 1.4. The student correctly determines a total profit of $70.28.

2 points: Partial understanding of numbers and operations and the number system. The student makes an error in 1 of the 3 sections of Part D.

1 point: Limited understanding of numbers and operations and the number system. The student makes an error in 2 of the 3 sections of Part D.

0 points: Little or no understanding of numbers and operations and the number system. The student makes errors in all 3 sections of Part D.