Sacramento City Unified School District

CCSS-M Teacher Professional Learning

Session #1, October 2014

Grade 7

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(Selected pages relevant to session work)

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Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*
2. Recognize and represent proportional relationships between quantities.
    a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
    b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
    c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.
    d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
    a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
    b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
    c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
    d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
    a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
    b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

Expressions and Equations 7.EE

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
   b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Geometry 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Common Core State Standards - Mathematics
Standards for Mathematical Practices – 7th Grade

Standard for Mathematical Practice	7th Grade
1: Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, -What is the most efficient way to solve the problem?, -Does this make sense?, and -Can I solve the problem in a different way?

2: Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3: Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like -How did you get that?, -Why is that true? -Does that always work? They explain their thinking to others and respond to others' thinking.

4: Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

5: Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.
6: Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.

7: Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2 (3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.

8: Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

multiplication and division of signed numbers (**7.NS.2▲**). Sufficient practice is required so that students can compute sums and products of rational numbers in all cases, and also apply these concepts reliably to real-world situations.

The Number System	7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
    a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
    b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real world contexts.
    c. Apply properties of operations as strategies to multiply and divide rational numbers.
    d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
3. Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

Students continue to develop their understanding of operations with rational numbers by seeing that multiplication and division can be extended to signed rational numbers (**7.NS.2▲**). For instance, students can understand that in an account balance model, $(-3)(\$40.00)$ can be thought of as a record of 3 groups of debits (indicated by the negative sign) of $\$40.00$ each, resulting in a total contribution to the balance of $-\$120.00$. In a vector model, students can interpret the expression $(2.5)(-7.5)$ as the vector that points in the same direction as the vector representing -7.5, but is 2.5 times as long. Interpreting multiplication of two negatives in everyday terms can be troublesome, since negative money cannot be withdrawn from a bank. In a vector model, multiplying by a negative number reverses the direction of the vector (in addition to any stretching or compressing of the vector). Division is often difficult to interpret in

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

everyday terms as well, but can always be understood mathematically in terms of multiplication, specifically as multiplying by the reciprocal.

Multiplication of Signed Rational Numbers

In general, multiplication of signed rational numbers is performed as with fractions and whole numbers, but according to the following rules for determining the sign of the product:

$$\text{(i)} \quad (-a) \times b = -ab,$$

$$\text{(ii)} \quad (-a) \times (-b) = ab.$$

In these equations, both a and b can be positive, negative, or zero. Of particular importance is that $-1 \cdot a = -a$, that is, multiplying a number by negative one gives the opposite of the number. The first of these rules can be understood in terms of models as mentioned above. The second can be understood as being a result of properties of operations (refer to “A Derivation of the Fact that $(-1)(-1) = 1$” below). Students can also become comfortable with rule (ii) by examining patterns in products of signed numbers, such as in the table below, though this does not constitute a valid mathematical proof.

Example: Using Patterns to Investigate Products of Signed Rational Numbers.

Students can investigate a table like the one below. It is natural to conjecture that the missing numbers in the table should be $5, 10, 15,$ and 20 (reading them from left to right).

5×4	5×3	5×2	5×1	5×0	$5 \times (-1)$	$5 \times (-2)$	$5 \times (-3)$	$5 \times (-4)$
20	15	10	5	0	-5	-10	-15	-20
$(-5) \times 4$	$(-5) \times 3$	$(-5) \times 2$	$(-5) \times 1$	$(-5) \times 0$	$(-5) \times (-1)$	$(-5) \times (-2)$	$(-5) \times (-3)$	$(-5) \times (-4)$
-20	-15	-10	-5	0	$??$	$??$	$??$	$??$

Ultimately, if students come to an understanding that $(-1)(-1) = 1$, then the fact that $(-a)(-b) = ab$ follows immediately using the associative and commutative properties of multiplication:

$$(-a)(-b) = (-1 \cdot a)(-1 \cdot b) = (-1)a(-1)b = (-1)(-1)ab = 1 \cdot ab = ab.$$

After arriving at a general understanding of these two rules for multiplying signed numbers, students can multiply any rational numbers by finding the product of the absolute values of the numbers and then determining the sign according to the rules.

[Note: Sidebar]

A Derivation of the Fact that $(-\mathbf{1})(-\mathbf{1}) = \mathbf{1}$.

Students are reminded that addition and multiplication are related by a very important algebraic property, the *distributive property of multiplication over addition:*

$$a(b + c) = ab + ac,$$

valid for all numbers a, b and c. This property plays an important role in the derivation here as it does in all of mathematics. The basis of this derivation is that the *additive inverse* of the number -1 (that is, the number you add to -1 to obtain 0) is equal to 1. We observe that if we add $(-1)(-1)$ and (-1), the distributive property reveals something interesting:

$$(-1)(-1) + (-1) = (-1)(-1) + (-1)(1) \quad \text{(since } (-1) = (-1)(1)\text{)}$$

$$= (-1)[(-1) + 1] \quad \text{(by the distributive property)}$$

$$= (-1) \cdot 0 = 0 \quad \text{(since } (-1) + 1 = 0.\text{)}$$

Thus, when adding the quantity $(-1)(-1)$ to -1, the result is 0. This implies that $(-1)(-1)$ is none other than the additive inverse of -1, or in other words, 1. This completes the derivation.

Division of Rational Numbers

The relationship between multiplication and division allows students to infer the sign of the quotient of two rational numbers. Otherwise, division is performed as usual with whole numbers and fractions, with the sign to be determined.

Examples: Determining the Sign of a Quotient.

If $x = (-16) \div (-5)$, then $x \cdot (-5) = -16$. It follows that whatever the value of x is it must be a positive

number. In this case, $x = \frac{-16}{-5} = \frac{16}{5}$. This line of reasoning can be used to justify the general fact that for rational numbers p and q, with $q \neq 0$, $\frac{-p}{-q} = \frac{p}{q}$.
If $y = \frac{-0.2}{50}$, then $y \cdot 50 = -0.2$. This implies that y must be negative, and we have that $y = \frac{-0.2}{50} = -\frac{2}{500} = -\frac{4}{1000} = -0.004$.
If $z = \frac{0.2}{-50}$, then $z \cdot (-50) = 0.2$. This implies that z must be negative, and we have that $z = \frac{0.2}{-50} = -\frac{2}{500} = -\frac{4}{1000} = -0.004$.

The latter two examples above show that $\frac{-0.2}{50} = \frac{0.2}{-50}$. In general, it is true that $\frac{-p}{q} = \frac{p}{-q}$ for rational numbers (with $q \neq 0$). Students often have trouble interpreting the expression $-\left(\frac{p}{q}\right)$. To begin with, we should interpret this as meaning "the opposite of the number $\frac{p}{q}$." Considering a specific example, notice that since $-\left(\frac{5}{2}\right) = -(2.5)$ is a negative number, the product of 4 and $-\left(\frac{5}{2}\right)$ must also be a negative number. We determine that $4 \cdot \left(-\left(\frac{5}{2}\right)\right) = -10$. On the other hand, this equation implies that $-\left(\frac{5}{2}\right) = -10 \div 4$, in other words, that $-\left(\frac{5}{2}\right) = \frac{-10}{4} = \frac{-5}{2}$. A similar line of reasoning shows that $-\left(\frac{5}{2}\right) = \frac{5}{-2}$. Examples like these help justify that $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$ **(7.NS.2.b▲)**.[7]

Students solve real-world and mathematical problems involving positive and negative rational numbers while learning to compute sums, differences, products, and quotients of rational numbers. They also come to understand that every rational number can be written as a decimal with an expansion that eventually repeats or terminates (i.e., eventually repeats with 0s). **(7.NS.2c-d, 7.NS.3▲)** (**MP.1, MP.2, MP.5, MP.6, MP.7, MP.8**)

Examples of Rational Number Problems.
1. During a business call, Marion was told of the most recent transactions in the business account.

[7] Incidentally, this also shows why it is unambiguous to write $-\frac{p}{q}$, and drop the parenthesis.

There were deposits of $\$1{,}250$ and $\$3040.57$, three withdrawals of $\$400$, and the bank removed two penalties to the account of $\$35$ that were the bank's errors. How much did the balance of the account change based on this report?

Solution: The deposits are considered positive changes to the account, the three withdrawals are considered negative changes, and the two removed penalties of $\$35$ can be considered as subtracting debits to the account. One might represent the total change to the balance as:

$$\$1{,}250.00 + \$3{,}040.57 - 3(\$400.00) - 2(-\$35.00) = \$3{,}160.57.$$

Thus, the balance of the account increased altogether by $\$3{,}160.57$

2. Find the product $(-373) \cdot 8$.

Solution: "I know that the first number has a factor of (-1) in it, so that the product will be negative. So now I just need to find $373 \cdot 8 = 2400 + 560 + 24 = 2984$. So $(-373) \cdot 8 = -2984$."

3. Find the quotient $\left(-\frac{25}{28}\right) \div \left(-\frac{5}{4}\right)$.

Solution: "I know that the result is a positive number. This looks like a problem where I can just divide numerator and denominator: $\frac{25}{28} \div \frac{5}{4} = \frac{25 \div 5}{28 \div 4} = \frac{5}{7}$. The quotient is $\frac{5}{7}$."

4. Represent each problem by a diagram, a number line, and an equation. Solve each problem. (a) A weather balloon is 100,000 feet above sea level, and a submarine is 3 miles below sea level. How far apart are the submarine and the weather balloon? (b) John was \$3.75 in debt, and Mary was \$0.50 ahead. John found an envelope with some money in it, and after that he had the same amount of money as Mary. How much was in the envelope?

Domain: Expressions and Equations

In grade six students began the study of equations and inequalities and methods for solving them. In grade seven students build on this understanding and use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. Students also work toward fluently solving equations of the form $px + q = r$ and $p(x + q) = r$.

Expressions and Equations	7.EE
Use properties of operations to generate equivalent expressions.	
1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	

Domain: The Number System (NS)

Cluster: *Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*

Standard: **Grade 7.NS.2**

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $-1 \times -1 = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $\left(-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}\right)$. Interpret quotients of rational numbers by describing real-world contexts.
- **c.** Apply properties of operations as strategies to multiply and divide rational numbers.
- **d.** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.6 Attend to precision

Connections: See **Grade 7.NS.1**

Explanations and Examples:

Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign. Using long division from elementary school, students understand the difference between terminating and repeating decimals. This understanding is foundational for work with rational and irrational numbers in 8^{th} grade. For example, identify which fractions will terminate (the denominator of the fraction in reduced form only has factors of 2 and/or 5.)

Multiplication and division of integers is an extension of multiplication and division of whole numbers.

Examples

Examine the family of equations. What pattern do you see?
Create a model and context for each of the products.
Write and model the family of equations related to $2 \times 3 = 6$.

Equation	Number Line Model	Context
$2 \times 3 = 6$	0 3 6	Selling two packages of apples at $3.00 per pack
$2 \times -3 = -6$	-6 -3 0	Spending 3 dollars each on 2 packages of apples
$-2 \times 3 = -6$	-6 -4 -2 0	Owing 2 dollars to each of your three friends
$-2 \times -3 = 6$	0 2 4 6	Forgiving 3 debts of $2.00 each

Instructional Strategies:

Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers. For example, beginning with known facts, students predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2 and 3 below).

Table 1	Table 2	Table 3
4 x 4 = 16	4 x 4 = 16	-4 x -4 = 16
4 x 3 = 12	4 x 3 = 12	-4 x -3 = 12
4 x 2 = 8	4 x 2 = 8	-4 x -2 = 8
4 x 1 = 4	4 x 1 = 4	-4 x -1 = 4
4 x 0 = 0	4 x 0 = 0	-4 x 0 = 0
4 x -1 =	-4 x 1 =	-1 x - 4 =
4 x - 2 =	-4 x 2 =	-2 x - 4 =
4 x - 3 =	-4 x 3 =	-3 x - 4 =
4 x - 4 =	-4 x 4 =	-4 x - 4 =

Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers ($-4 \times -4 = 16$, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products and communtativity of multiplication. Then students should be asked to answer these questions and prove their responses:

- Is it always true that multiplying a negative factor by a positive factor results in a negative product?
- Does a positive factor times a positive factor always result in a positive product?
- What is the sign of the product of two negative factors?
- When three factors are multiplied, how is the sign of the product determined?

Major Supporting Additional **Depth Opportunities(DO)**

- How is the numerical value of the product of any two numbers found?

Students can use number lines with arrows and hops, groups of colored chips or logic to explain their reasoning. When using number lines, establishing which factor will represent the length, number and direction of the hops will facilitate understanding. Through discussion, generalization of the rules for multiplying integers would result.

Division of integers is best understood by relating division to multiplication and applying the rules. In time, students will transfer the rules to division situations. (Note: In 2b, this algebraic language $\left(-\left(\frac{p}{q}\right)=\frac{(-p)}{q}=\frac{p}{(-q)}\right)$ is written for the teacher's information, not as an expectation for students.)

Ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with fractions and decimals.

In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected.

Commented [MAF3]: This is identical to the standard before this one is this correct?

Major Supporting Additional **Depth Opportunities(DO)**

Domain: The Number System (NS)

Cluster: *Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*

Standard: Grade 7.NS.3

Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Suggested Standards for Mathematical Practice (MP):

MP.1 Make sense of problems and persevere in solving them.

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 7.NS.2**

Explanations and Examples:

Students use order of operations from 6th grade to write and solve problem with all rational numbers.

Examples:

Your cell phone bill is automatically deducting $32 from your bank account every month.
How much will the deductions total for the year?

$$32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 = 12(-32)$$

It took a submarine 20 seconds to drop to 100 feet below sea level from the surface.
What was the rate of the descent?

$$\frac{-100\ feet}{20\ seconds} = \frac{-5\ feet}{1\ second} = -5\ ft/sec$$

The three seventh grade classes at Sunview Middle School collected the most box tops for a school fundraiser, and so they won a $600 prize to share between them. Mr. Aceves' class collected 3,760 box tops, Mrs. Baca's class collected 2,301, and Mr. Canyon's class collected 1,855. How should they divide the money so that each class gets the same fraction of the prize money as the fraction of the box tops that they collected?

A teacher might start out by asking questions like, "Which class should get the most prize money?
Should Mr. Aceves' class get more or less than half of the money?
Mr. Aceves' class collected about twice as many box tops as Mr. Canyon's class - does that mean that
Mr. Aceves' class will get about twice as much prize money as Mr. Canyon's class?"

This task represents an opportunity for students to engage in Standard MP.5 *Use appropriate tools strategically.* There is little benefit in students doing the computations by hand (few adults would), and so provides an opportunity to discuss the value of having a calculator and when it is (and is not) appropriate to use it.

Sample Solution:
All together, the students collected $3{,}750 + 2{,}301 + 1{,}855 = 7{,}916$ box tops.
Mr. Aceves' class collected $\frac{3760}{7916}$of the box tops.
The amount for Mr. Aceves' class is $\left(\frac{3760}{7916}\right) 600 \approx 284.99$
Mrs. Baca's class collected $\frac{2301}{7916}$of the box tops.
The amount for Mrs. Baca's class is $\left(\frac{2301}{7916}\right) 600 \approx 174.41$
Mr. Canyon's class collected $\frac{1855}{7916}$of the box tops.
The amount for Mr. Canyon's class is $\left(\frac{1855}{7916}\right) 600 \approx 140.60$
$284.99 should go to Mr. Aceves' class, $174.41 should go to Mrs. Baca's class, and $140.60 should go to Mr. Canyon's class.

Instructional Strategies: See **Grade 7.NS. 1-2**
See Also: Number Systems (Grade 6-8) and Number High School

Illustrative math-"Sharing Prize Money":

For detailed information, see Progressions for the Common Core State Standards in Mathematics: Number System 6-8.

SCUSD 7th Grade Curriculum Map

Unit 4: Operations of Rational Numbers – Multiplication and Division
Sequence of Learning Outcomes **7.NS.2 and 7.NS.3**
1) Understand and develop fluency of multiplication of integers through definition of integers and multiplication as repeated addition. Additional methods that should be explored include using patterns in products of integers and the proof of why (-1)(-1) = 1. (*CA Framework* p. 25, 26) 7.NS.2
2) Develop the rules for multiplying integers and extend that understanding to all rational numbers for the purpose of fluency. Apply rules of signed numbers to real-world contexts. 7.NS.2
3) Extend the rules of multiplication to division of integers using the inverse relationship between multiplication and division. Apply division of integers to real-world contexts. 7.NS.2
4) Apply rules of multiplication and division to all rational numbers. Solve real-world problems involving both operations. 7.NS.2
5) Convert rational numbers to decimals using long division; know that the decimal form of a rational number terminates in 0's or repeats. 7.NS.2
6) Solve real-world and mathematical problems involving the four operations with rational numbers. (Framework p.28) 7.NS.2

Big Ideas Math Course 2

Chapters 1 and 2
Sequence of Learning Objectives **Lessons 1.1, 1.4, 1.5, 2.1 and 2.4**
Lesson 1.1 – Integers and Absolute Value In this lesson, you will • Define the absolute value of a number. • Find absolute values of numbers • Solve real-life problems Preparing for Standard 7.NS.1, 7.NS.2, and 7.NS.3
Lesson 1.4 – Multiplying Integers In this lesson, you will • Multiply integers • Solve real-life problems Learning 7.NS.2a, 7.NS.2c, and 7.NS.3
Lesson 1.5 – Dividing Integers In this lesson, you will • Divide integers • Solve real-life problems Learning 7.NS.2b and 7.NS.3
Lesson 2.1 – Rational Numbers In this lesson, you will • Understand that a rational number is an integer divided by an integer. • Convert rational numbers to decimals Learning Standards 7.NS.2b and 7.NS.2d
Lesson 2.4 – Multiplying and Dividing Rational Numbers In this lesson, you will • Multiply and divide rational numbers. • Solve real-life problems Learning Standards 7.NS.2a, 7.NS.2b, 7.NS.2c, 7.NS.3

2 Chapter Test

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Write the rational number as a decimal.

1. $\frac{7}{40}$
2. $-\frac{1}{9}$
3. $-\frac{21}{16}$
4. $\frac{36}{5}$

Write the decimal as a fraction or a mixed number in simplest form.

5. -0.122
6. 0.33
7. -4.45
8. -7.09

Add or subtract. Write fractions in simplest form.

9. $-\frac{4}{9}+\left(-\frac{23}{18}\right)$
10. $\frac{17}{12}-\left(-\frac{1}{8}\right)$
11. $9.2+(-2.8)$
12. $2.86-12.1$

Multiply or divide. Write fractions in simplest form.

13. $3\frac{9}{10}\times\left(-\frac{8}{3}\right)$
14. $-1\frac{5}{6}\div 4\frac{1}{6}$
15. $-4.4\times(-6.02)$
16. $-5\div 1.5$
17. $-\frac{3}{5}\bullet\left(2\frac{2}{7}\right)\bullet\left(-3\frac{3}{4}\right)$
18. $-6\bullet(-0.05)\bullet(-0.4)$

19. **ALMONDS** How many 2.25-pound containers can you make with 24.75 pounds of almonds?

20. **FISH** The elevation of a fish is −27 feet.

 a. The fish decreases its elevation by 32 feet, and then increases its elevation by 14 feet. What is its new elevation?

 b. Your elevation is $\frac{2}{5}$ of the fish's new elevation. What is your elevation?

21. **RAINFALL** The table shows the rainfall (in inches) for three months compared to the yearly average. Is the total rainfall for the three-month period greater than or less than the yearly average? Explain.

November	December	January
−0.86	2.56	−1.24

22. **BANK ACCOUNTS** Bank Account A has \$750.92, and Bank Account B has \$675.44. Account A changes by –\$216.38, and Account B changes by −\$168.49. Which account has the greater balance? Explain.

Test Item References

Chapter Test Questions	Section to Review	Common Core State Standards
1–8	2.1	7.NS.2b, 7.NS.2d
9, 11, 21, 22	2.2	7.NS.1a, 7.NS.1b, 7.NS.1d, 7.NS.3
10, 12, 20(a)	2.3	7.NS.1c, 7.NS.1d, 7.NS.3
13–19, 20(b)	2.4	7.NS.2a, 7.NS.2b, 7.NS.2c, 7.NS.3

Test-Taking Strategies

Remind students to quickly look over the entire test before they start so that they can budget their time. On tests, it is really important for students to **Stop** and **Think.** When students hurry on a test dealing with signed numbers, they often make "sign" errors. Sometimes it helps to represent each problem with a number line to ensure that they are thinking through the process.

Common Errors

- **Exercises 1–4** Students may forget to carry the negative sign through the division operation. Tell them to create a space for the final answer and to write the sign of the number in the space at the beginning.
- **Exercises 9 and 10** Students may forget to find a common denominator. Remind students that adding and subtracting fractions always requires a common denominator.
- **Exercise 14** Students may use the reciprocal of the first fraction instead of the second, or they might forget to write a mixed number as an improper fraction before finding the reciprocal. Review multiplying and dividing fractions and the definition of reciprocal.
- **Exercises 15 and 16** Students may place the decimal point incorrectly in their answers. Remind students of the rules for multiplying and dividing decimals. Also, remind students to use estimation to check their answers.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials *BigIdeasMath.com* Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start Standards Assessment

Answers

1. 0.175
2. $-0.\overline{1}$
3. -1.3125
4. 7.2
5. $-\frac{61}{500}$
6. $\frac{33}{100}$
7. $-4\frac{9}{20}$
8. $-7\frac{9}{100}$
9. $-1\frac{13}{18}$
10. $1\frac{13}{24}$
11. 6.4
12. -9.24
13. $-10\frac{2}{5}$
14. $-\frac{11}{25}$
15. 26.488
16. $-3.\overline{3}$
17. $5\frac{1}{7}$
18. -0.12
19. 11 containers
20. a. -45 feet

    b. -18 feet
21. greater than; The sum of the three months is 0.46.
22. Bank Account A; Bank Account A has \$534.54 while Bank Account B only has \$506.95.

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Illustrative Mathematics

7.NS Equivalent fractions approach to non-repeating decimals

Alignments to Content Standards

- Alignment: 7.NS.A.2.d

Tags

- *This task is not yet tagged.*

Malia found a "short cut" to find the decimal representation of the fraction $\frac{117}{250}$. Rather than use long division she noticed that because $250 \times 4 = 1000$,

$$\frac{117}{250} = \frac{117 \times 4}{250 \times 4} = \frac{468}{1000} = 0.468.$$

a. For which of the following fractions does Malia's strategy work to find the decimal representation?

$$\frac{1}{3}, \frac{3}{4}, -\frac{6}{25}, \frac{18}{7}, \frac{13}{8} \quad \text{and} \quad -\frac{113}{40}.$$

For each one for which the strategy does work, use it to find the decimal representation.

b. For which denominators can Malia's strategy work?

Commentary

This task is most suitable for instruction. The purpose of the task is to get students to reflect on the definition of decimals as fractions (or sums of fractions), at a time when they are seeing them primarily as an extension of the base-ten number system and may have lost contact with the basic fraction meaning. Students also have their understanding of equivalent fractions and factors reinforced.

If students need help connecting this method with that of long division, they can be asked to perform long division when the denominator is a power of ten.

The denominators identified in the second part, namely numbers which are factors of powers of ten (or, equivalently, numbers for which 2 and 5 are the only prime factors) are in fact the only ones whose decimal expansions terminate when the fraction is in reduced form. While the problem does not ask for this fact, it should be shared and can be explained readily: A terminating decimal is equal to $\frac{a}{10^n}$. In reduced form, the denominator must be a quotient - and thus a factor - of 10^n.

Solutions

Solution: Solution

a.
- The strategy does not work for $\frac{1}{3}$ because there are no multiples of 3 which are powers of 10.
- Because $4 \times 25 = 100$, $\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$.
- $-\frac{6}{25} = -\frac{24}{100} = -0.24$.
- The strategy does not work for $\frac{18}{7}$ because there are no multiples of 7 which are powers of 10.
- $\frac{13}{8} = \frac{13 \times 125}{8 \times 125} = \frac{1625}{1000} = 1.625.$
- $-\frac{113}{40} = -2\frac{37}{40} = -2 + (-\frac{37 \times 25}{40 \times 25}) = -2 + (-\frac{825}{1000}) = -2.825$

b. The strategy can work for any denominator which is a factor of a power of 10. In this case one can multiply the numerator and denominator by the complementary factor (that is, the quotient of that power of 10 by the denominator) to obtain a fraction with denominator equal to that power of 10. Such fractions are represented by terminating decimals.