CCSS-M Teacher Professional Learning

Session #1, October 2014

Grade 6

Packet Contents

(Selected pages relevant to session work)

Content Standards

Standards for Mathematical Practice

California Mathematical Framework

Kansas CTM Flipbook

Learning Outcomes

Sample Assessment Items

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
    a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
    b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
    c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
7. Understand ordering and absolute value of rational numbers.
    a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
    b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*
    c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
    d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
    a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
    b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.

3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Geometry 6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Standard for Mathematical Practice	6th Grade
1: Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, -What is the most efficient way to solve the problem?, -Does this make sense?, and -Can I solve the problem in a different way?

2: Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3: Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like -How did you get that?, -Why is that true? -Does that always work? They explain their thinking to others and respond to others' thinking.

4: Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

5: Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6: Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.

7: Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2 (3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.

8: Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Expressions and Equations 6.EE

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

In the elementary grades students explored the concept of equality. In sixth grade students explore equations as one expression being set equal to a specific value. A solution is a value of the variable that makes the equation true. Students use various processes to identify such value(s) that when substituted for the variable will make the equation true (**6.EE.5▲**). Students can use manipulatives and pictures (e.g., tape-like diagrams) to represent equations and their solution strategies. When writing equations, students learn to be precise in their definition of a variable, e.g., writing "n equals John's age in years" as opposed to simply writing "n is John." (**6.EE.6▲**). (MP.6)

Examples: Solving Equations of the Form $p + x = q$ and $px = q$. (**6.EE.7▲**).

1. Joey had 26 game cards. His friend Richard gave him some more and now he has 100 cards. How many cards did his friend Richard give him? Write and equation and solve your equation.

Solution: Since Richard gave him some more cards, we let n be the number of cards that Richard gave Joey. This means he now has $26 + n$ cards. But the number of cards Joey has is 100, so we get the equation $26 + n = 100$. Using the relationship between addition and subtraction, we see that $n = 100 - 26 = 74$, which means that his friend gave him 74 cards. One can represent this equation with a tape-like diagram:

100	
26	n

2. A book of Theme Park Ride tickets costs $30.00. Each ticket on its own costs $2.50. How many tickets come in each book? Write and solve an equation that represents this situation.

Solution: If s represents the number of stamps in one booklet, then $(2.50)s$ is the cost of s tickets in dollars. Since the cost of one book is $\$30.00$, solving the equation $(2.50)s = 30.00$ would give the number of tickets. To solve this, we realize that if $2.50 \times s = 30.00$, then $s = 30.00 \div 2.50 = 12$. This means there are 12 tickets in each book.

3. Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an equation that represents this situation and solve it to determine how much one pair of jeans cost.

$56.58		
J	J	J

Solution: If J represents the cost of one pair of jeans in dollars, then the equation becomes $3J = 56.58$. If we solve this for J, we find $J = 56.58 \div 3 = 18.86$. This means each pair of jeans cost $18.86.

4. Julio gets paid $20 for babysitting. He spent $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julio has left.

Solution: One equation might be $1.99 + 6.50 + x = 20.00$, where x represents how many dollars he has left. We find that $x = 11.51$, so that he has $\$11.51$ left.

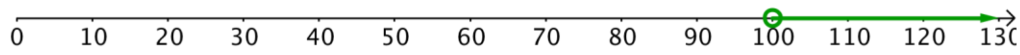
(Adapted from Arizona 2012, N. Carolina 2012, and KATM 6th FlipBook 2012)

Many real-world situations are represented by inequalities. In grade six, students write simple inequalities involving < or > to represent real world and mathematical situations, and they use the number line to model the solutions (**6.EE.8▲)**. Students learn that when representing inequalities of these forms on a number line, the common practice is to draw an arrow on or above the number line with an open circle on or above the number in the inequality. The arrow indicates the numbers greater than or less than the number in question, and that the solutions extend indefinitely. The arrow is a solid line, to indicate that even fractional and decimal amounts (i.e. points between dashes on the line) are included in the solution set.

Examples: Inequalities of the Form $x < c$ and $x > c$.

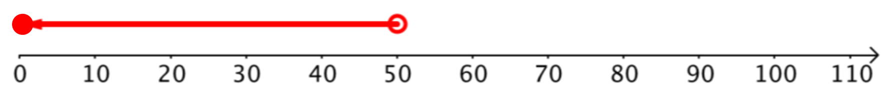
1. The class must raise more than $100 to go on the field trip. Let m represent the amount of money in dollars that the class raises. Write an inequality that describes how much the class needs to raise. Represent this on a number line.

Solution: Since the amount of money, m, needs to be greater than 100, the inequality is $m > 100$. A number line diagram for this might look like:



2. The Flores family spent less than \$50.00 on groceries last week. Write an inequality that describes this situation and graph the solution on a number line.

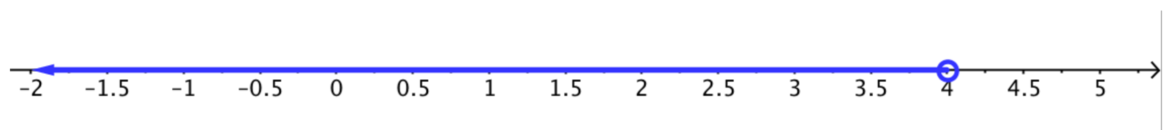
Solution: If we let g represent the amount of money in dollars the family spent on groceries last week, then the inequality becomes $k < 50$. We might represent this in the following way:



(In this example, it doesn't make sense that the Flores family could have spent a negative amount of dollars on groceries, so the arrow would stop precisely at \$0; we would typically represent this with a dot over 0 rather than the arrow.)

3. Graph $x < 4$.

Solution: This represents all numbers less than 4:



(Adapted from Arizona 2012, N. Carolina 2012, and KATM 6th FlipBook 2012)

Expressions and Equations 6.EE

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

In grade six students investigate the relationship between two variables, beginning with the distinction between dependent and independent variables (**6.EE.9▲**). The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is

Domain: Expressions and Equations (EE)

Cluster: Reason about and solve one-variable equations and inequalities.

Standard: **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Standards for Mathematical Practice (MP):

MP 2 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

Connections:

This cluster is connected to the Grade 6 Critical Area of Focus #3, **Writing, interpreting and using expressions, and equations.**

Explanations and Examples:

Beginning experiences in solving equations should require students to understand the meaning of the equation as well as the question being asked. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies such as using reasoning, fact families, and inverse operations. Students may use balance models in representing and solving equations and inequalities.

Consider the following situation: Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?
This situation can be represented by the equation $26 + n = 100$ where n is the number of papers the teacher gives to Joey. This equation can be stated as "some number was added to 26 and the result was 100". Students ask themselves "What number was added to 26 to get 100?" to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem.

- Reasoning: 26 + 70 is 96. 96 + 4 is 100, so the number added to 26 to get 100 is 74.
- Use knowledge of fact families to write related equations:
  $n + 26 = 100$, $100 - n = 26$, $100 - 26 = n$. Select the equation that helps you find n easily.
- Use knowledge of inverse operations: Since subtraction "undoes" addition then subtract 26 from 100 to get the numerical value of n
- Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.

100	
26	*n*

Examples continued next page

Examples:

- The equation $0.44s = 11$ where *s* represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies you used to determine your answer. Show that your solution is correct using substitution.
- Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement?

Students identify values from a specified set that will make an equation true. For example, given the expression *x* + 2 1/2 which of the following value(s) for *x* would make *x* + 2 ½ = 6.

{0, 3 ½, 4}

By using substitution, students identify 3 ½ as the value that will make both sides of the equation equal.

The solving of inequalities is limited to choosing values from a specified set that would make the inequality true. For example, find the value(s) of *x* that will make *x* + 3.5 ≥ 9.

{5, 5.5, 6, 15/2, 10.2,15}

Using substitution, students identify 5.5, 6, 15/2,10.2, and 15 as the values that make the inequality true. NOTE: If the inequality had been *x* + 3.5 > 9, then 5.5 would not work since 9 is not greater than 9.

This standard is foundational to **6.EE.7** and **6.EE.8**

<u>Instructional Strategies</u> EE. 5-8

In order for students to **understanding** equations: The skill of solving an equation must be developed *conceptually* before it is developed *procedurally*. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. For example, in the equation *x* + 21 = 32 students know that 21 + 9 = 30 therefore the solution must be 2 more than 9 or 11, so *x* = 11.

Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both types of equations (single solution for the situation) and inequalities (multiple solutions for the situation). Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is being introduced in Grade 6. When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers but includes rational numbers. This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. As an extension to this concept, certain situations may require a solution between two numbers. For example, a problem situation may have a solution that requires more than 10 but not greater than 25. Therefore, the exploration with students as to what this would look like both on a number line and symbolically is a reasonable extension.

The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations/inequalities for a situation. This process should go both ways; Students should be able to write a mathematical phrase for an equation. Additionally, the writing of equations from a situation or story does not come naturally for many students. A strategy for assisting with this is to give students an equation and ask them to come up with the situation/story that the equation could be referencing.

Domain: **Expressions and Equations (EE)**

Cluster: Reason about and solve one-variable equations and inequalities.

Standard: **6.EE.6**
Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Standards for Mathematical Practice (MP):

MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 6 Attend to precision.
MP 7 Look for and make use of structure.

Connections:

See EE.5

Explanations and Examples:

6.EE.6. Students write expressions to represent various real-world situations. For example, the expression $a + 3$ could represent Susan's age in three years, when a represents her present age. The expression $2n$ represents the number of wheels on any number of bicycles. Other contexts could include age (Johnny's age in 3 years if a represents his current age) and money (value of any number of quarters)

Given a contextual situation, students define variables and write an expression to represent the situation. For example, the skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people.

N = the number of people

100 + 5n

Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Examples:

- Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.
  (Solution: $2c + 3$ where c represents the number of crayons that Elizabeth has.)
- An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent.
  (Solution: $28 + 0.35t$ where t represents the number of tickets purchased)
- Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned.
  (Solution: $15h + 20 = 85$ where h is the number of hours worked)
- Describe a problem situation that can be solved using the equation
  $2c + 3 = 15$; where c represents the cost of an item
- Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. (Solution: $5.00 + *n*)

Instructional Strategies

See EE.5

Common Misconceptions:

See .EE.5

				6.EE.6	

Domain: **Expressions and Equations (EE)**

Cluster: Reason about and solve one-variable equations and inequalities.

Standard: **6.EE.7**
Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Standards for Mathematical Practice (MP):
MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

Connections:
See EE.5

Explanations and Examples:
6.EE.7 Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, $x + 4$, any value can be substituted for the x to generate a numerical answer; however, in the equation $x + 4 = 6$, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable.
Students write equations from real-world problems and then use inverse operations to solve one-step equations. Equations may include fractions and decimals with non-negative solutions.

Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies.

Example:

- Meagan spent \$56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

\$56.58		
J	J	J

Sample Solution: Students might say: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J = \$56.58$. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than \$10 each because 10 x 3 is only 30 but less than \$20 each because 20 x 3 is 60. If I start with \$15 each, I am up to \$45. I have \$11.58 left. I then give each pair of jeans \$3. That's \$9 more dollars. I only have \$2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another \$0.86. Each pair of jeans costs \$18.86 (15+3+0.86). I double check that the jeans cost \$18.86 each because \$18.86 x 3 is \$56.58."

Continued on next page

- Julio gets paid \$20 for babysitting. He spends \$1.99 on a package of trading cards and \$6.50 on lunch. Write and solve an equation to show how much money Julio has left. (Solution: $20 = 1.99 + 6.50 + x$, $x = \$11.51$)

20		
1.99	6.50	money left over (m)

Instructional Strategies:
See EE.5

Common Misconceptions:
See EE.5

Domain: **Expressions and Equations (EE)**

Cluster: Reason about and solve one-variable equations and inequalities.

Standard: **6.EE.8**

Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Standards for Mathematical Practice (MP):

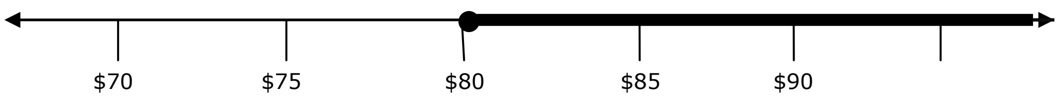
MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

Connections:

See EE.5

Explanations and Examples:

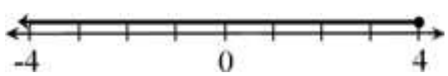
6.EE.8 Many real-world situations are represented by inequalities. Students write an inequality and represent solutions on a number line for various contextual situations.
For example, the class must raise at least \$80 to go on the field trip. If *m* represents money, then the inequality $m \geq$ to \$80. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.



A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions.

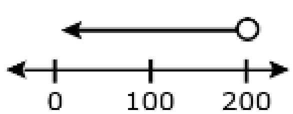
Examples:

- Graph $x \leq 4$.



- Jonas spent more than \$50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line.
- Less than \$200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality on a number line.

Solution: $200 > x$



Instructional Strategies & Common Misconceptions:

See EE.5

6.EE.8

Unit 4: Linear Relationships
Sequence of Learning Outcomes **6.EE.5, 6.EE.6, 6.EE.7, and 6.EE.8**
1) Represent real-world situations by writing expressions of the form x + p. Clearly define the meaning of the variable and the expression. 6.EE.6
2) Write and evaluate expressions representing real-world situations, of the form x + p, for multiple values of the variable. Use bar models and numeric representations. Define the meaning of the variable and expression. 6.EE.6
3) Write equations in the form x + p = q and create bar models to represent real-world situations. Clearly define the meaning of the variable and the both expressions in the equations. 6.EE.7
4) Solve equations of the form x + p = q using bar models and tables to facilitate guess and check. Use substitution to prove that a solution makes the equation true. 6.EE.5 and 6.EE.7
5) Solve equations of the form x + p = q using inverse operations. Use substitution to prove that a solution makes the equation true. 6.EE.5 and 6.EE.7
6) Represent real-world situations by writing expressions of the form px. Clearly define the meaning of the variable and the expression. 6.EE.6
7) Write and evaluate expressions representing real-world situations, of the form px, for multiple values of the variable. Use bar models and numeric representations. Define the meaning of the variable and expression. 6.EE.6
8) Write equations in the form px = q and create bar models to represent real-world situations. Clearly define the meaning of the variable and the both expressions in the equations. 6.EE.5 and 6.EE.7
9) Solve equations of the form px = q using bar models and tables to facilitate guess and check. Use substitution to prove that a solution makes the equation true. 6.EE.5 and 6.EE.7
10) Solve equations of the form px = q using inverse operations, where p is a whole number and then a fraction. Use substitution to prove that a solution makes the equation true. 6.EE.5 and 6.EE.7
11) Solve equations of the form px = q using inverse operations, where p is a decimal. Use

substitution to prove that a solution makes the equation true. 6.EE.5 and 6.EE.7
12) Distinguish between real-world situations represented by p + x = q and px = q, solving problems of each type using inverse operations. 6.EE.7
13) Write inequalities to represent real-world situations and identify possible solutions, recognizing that there can be infinitely many solutions. 6.EE.5 and 6.EE.8
14) Represent inequalities on a number line numerically and in real-world situations, representing constraints appropriately. 6.EE.5 and 6.EE.8

EnVision 6th Grade

Chapter 1: Variables and Expressions
Sequence of Learning Objectives **Lessons 1.6**
Lesson 1.6 - Evaluating Numerical Expressions In this lesson, you will : • Understand that variables represent unknown numbers • Write expressions utilizing variables 6.EE.6
Chapter 4: Achieving Fluency: **Adding, Subtracting, and Multiplying Decimals**
Sequence of Learning Objectives **Lessons 4.6 and 4.3**
Lesson 4.6 – Make a Table and Look for a Pattern In this lesson, you will: • Solve problems with models and equations 6.EE.6
Chapter 2:
Sequence of Learning Objectives **Lessons 2.1 – 2.9**
Lesson 2.1 – Understanding Equations In this lesson, you will: • Understand how to determine whether a given number makes an equation true Lesson 2.3 – Solving Addition and Subtraction Equations In this lesson, you will: • Understand how to get a variable alone 6.EE.5 & 6.EE.7
Lesson 2.9 – Draw a Picture and Write an Equation In this lesson, you will: • Solve problems with models and equations 6.EE.6
Lesson 2.4 – Draw a Picture and Write an Equation In this lesson, you will: • Use models and equations to solve problems Lesson 2.5 – Solving Multiplication and Division Equations In this lesson, you will: • Simplify equations to get a variable alone 6.EE.5 and 6.EE.7
Lesson 2.6 – Solving Equations with Fractions In this lesson, you will: • Use inverse relationships and properties of equality to isolate the variable 6.EE.7

Lesson 2.7 – Writing Inequalities
In this lesson, you will:

- Set up inequalities in mathematical and real-world conditions

6.EE.5 and 6.EE.8

Lesson 2.8 – Solving Inequalities
In this lesson, you will:

- Solve inequalities in mathematical and real-world inequalities

6.EE.5 and 6.EE.8

Name ____________________

Mark the best answer.

1. If $x - 3 = 10$, which of the following is also true? (2-2)

A $x - 3 + 3 = 10$

B $x - 3 + 3 = 10 - 10$

C $x - 3 - 3 = 10 + 3$

D $x - 3 + 3 = 10 + 3$

2. A symphony orchestra typically uses twice as many violins as cellos. If 36 violins are used, solve the equation $2x = 36$ to find the number of cellos used. (2-5)

A $x = 12$; There are 12 cellos used.

B $x = 18$; There are 18 cellos used.

C $x = 34$; There are 34 cellos used.

D $x = 72$; There are 72 cellos used.

3. Grant has \$7 in his pocket after spending \$8 at the movies. The equation shown can be used to find *d,* how many dollars he had before going to the movies. What step should be taken to get the variable alone? (2-3)
$d - 8 = 7$

A Add 8 to each side of the equation.

B Subtract 8 from each side of the equation.

C Multiply each side of the equation by 8.

D Divide each side of the equation by 8.

4. Which is a solution to the equation? (2-1)
$22 - x = 14$

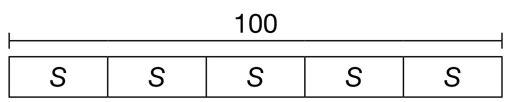
A 36

B 35

C 9

D 8

5. Five siblings buy a \$100 gift certificate for their parents and divide the cost equally. Which equation can be used to find *s,* the number of dollars each sibling pays? (2-9)



A $s + 5 = 100$

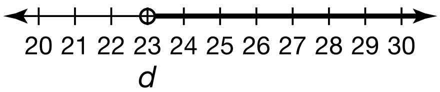
B $s - 5 = 100$

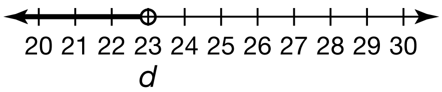
C $5s = 100$

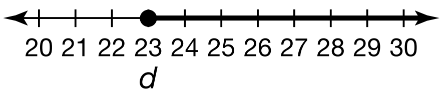
D $s \div 5 = 100$

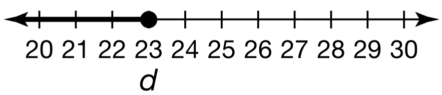
Name ______________________________

6. Which graph represents the solutions of the inequality $d < 23$? (2-8)

A


B


C


D


7. Which numbers make the inequality $x < 19$ true? Choose all numbers that apply. (2-8)

A 5

B 18

C 19

D 17

8. What value of p makes the following equation true? (2-5)
$p \div 15 = 3$

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9. Elizabeth is 64 inches tall, which is 9 inches taller than her brother Mac. Solve the equation $m + 9 = 64$ to find m, Mac's height in inches. (2-3)

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10. What step should be taken to get the variable alone in the equation shown? (2-5)
$7x = 84$

____

____

11. What value of x makes the following equation true? (2-3)
$x - 20 = 4$

____

12. Horace bought 84 pizzas for a school awards banquet. There will be 12 tables set up for the banquet, and Horace wants to divide the pizzas evenly between the tables. He thinks that each table will have 7 pizzas. Use the equation $84 \div p = 12$ to explain whether Horace is correct. (2-1)

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13. Kelly is on the rowing team. This week, Kelly's goal is to row 16 miles. So far, she has rowed $9\frac{3}{8}$ miles. Use the equation $9\frac{3}{8} + r = 16$ to find how many more miles Kelly needs to row. (2-6)

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Name ______________________________

14. Jose wrote that $9 + 9 = 18$. Then he wrote that $9 + 9 + n = 18 - n$. Are his equations balanced? Explain. (2-2)

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15. Solve. (2-6)

$c \times \frac{3}{4} = 2\frac{2}{5}$

____

16. The record for a city's greatest 1-day rainfall is 8.2 inches. Write an inequality to represent a rainfall that would beat this record. (2-7)

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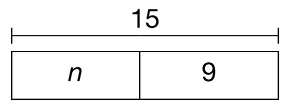
17. Jake finished 9 math problems on a homework assignment of 15 problems. Which equation can be used to find *n,* the number of problems Jake has left to do? (2-4)



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18. Dave is buying refreshments at the pool. After buying a smoothie for he and a friend for \$6.45, Dave is left with \$7.65. How much money did he begin with? Use the equation $x - \$6.45 = \$7.65$. (2-4)

____

19. Write an algebraic equation that represents the total weight (W) of eight boxes of blackberries, if b equals the weight of one box of blackberries. (2-5)

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Sample Assessment Questions

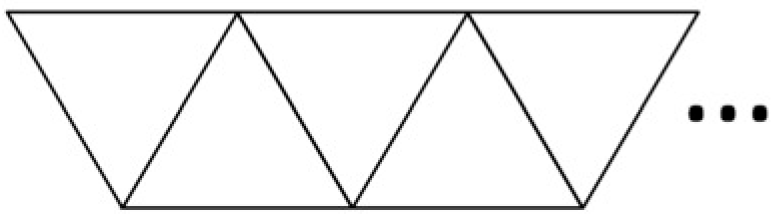
6.EE.5, 6.EE.6, 6.EE.7, and 6.EE.8

From www.IllustrativeMathematics.org

(linked from SCUSD Curriculum Map)

Triangle Tables

A classroom has triangular tables. There is enough space at each side of a table to seat one child. The tables in the class are arranged in a row (as shown in the picture below).



1. How many children can sit around 1 table? Around a row of two tables? Around a row of three tables?
2. Find an algebraic expression that describes the number of children that can sit around a row of *n* tables. Explain in words how you found your expression.
3. If you could make a row of 125 tables, how many children would be able to sit around it?
4. If there are 26 children in the class, how many tables will the teacher need to seat all the children around a row of tables?

Firefighter Allocation

A town's total allocation for firefighter's wages and benefits in a new budget is $600,000. If wages are calculated at $40,000 per firefighter and benefits at $20,000 per firefighter, write an equation whose solution is the number of firefighters the town can employ if they spend their whole budget. Solve the equation.

Pennies from Heaven

A penny is about 1/16 of an inch thick.

In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?

Log Ride

A theme park has a log ride that can hold 12 people. They also have a weight limit of 1500 lbs. per log for safety reason. If the average adult weighs 150 lbs., the average child weighs 100 lbs. and the log itself weights 200, the ride can operate safely if the inequality

$$150A + 100C + 200 \leq 1500$$

is satisfied (A is the number of adults and C is the number of children in the log ride together). There are several groups of children of differing numbers waiting to ride. Group one has 4 children, group two has 3 children, group three has 9 children, group four 6 children while group five has 5 children.

If 4 adults are already seated in the log, which groups of children can safely ride with them?